Intra-event correlations and the statistical moments of the identified particle multiplicity distributions in the RHIC beam energy scan data collected by STAR

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Measured “net-proton” and “net-charge” multiplicity distributions may provide insight on the conserved B and Q quantum numbers.

Measure the shapes of multiplicity distributions as quantified by the moments: $\mu$, $\sigma^2$, $S$, $K$

$S = \text{skewness}, K = \text{kurtosis}$

The products $S\sigma$ & $K\sigma^2$ are less volume dependent

Experimentally-measured moments products may be directly related to the susceptibility ratios (QCD order parameters) from the lattice theory. Values may relate to HG vs QGP phases...

In the NLSM, experimentally-measured moments products may also be proportional to powers of the correlation length. (critical opalescence) Divergent values may indicate the Critical Point...
No strong non-monotonicity seen, but there is an apparent dip at ~19.6-27 GeV

<table>
<thead>
<tr>
<th>√s_{NN} (GeV)</th>
<th>&lt;μ_B&gt;*</th>
</tr>
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<tbody>
<tr>
<td>7.7</td>
<td>421</td>
</tr>
<tr>
<td>11.5</td>
<td>316</td>
</tr>
<tr>
<td>19.6</td>
<td>206</td>
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<tr>
<td>27</td>
<td>156</td>
</tr>
<tr>
<td>39</td>
<td>112</td>
</tr>
<tr>
<td>62.4</td>
<td>73</td>
</tr>
<tr>
<td>200</td>
<td>24</td>
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</tbody>
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* Cleymans et al. PRC 73, 034905 (2006)
In this presentation, I will describe the comparison of the net-p and net-Q data to two data-driven techniques that explicitly break the intra-event correlations between $N_{\text{pos}}$ and $N_{\text{neg}}$.

- Do intra-event correlations between $N_{\text{pos}}$ and $N_{\text{neg}}$ affect the measured net-X moments?
- Can the net-X moments be understood from the $N_{\text{pos}}$ and $N_{\text{neg}}$ distributions alone?

“Independent Random Variable (IRV) Cumulant Arithmetic”

A feature of cumulants is their additivity for pairs of independent random variables.

$$C_k(u+v) = C_k(u) + C_k(v)$$

for net-X, i.e. “u-v” with $u=N_{\text{pos}}$ and $v=N_{\text{neg}}$,

$$C_k(u-v) = C_k(u) + (-1)^k \times C_k(v)$$

$$S\sigma = C_3/C_2 \quad \text{and} \quad K\sigma^2 = C_4/C_2$$ \hspace{1cm} (C_1=\text{mean, } C_2=\text{variance})

“Sampled Singles”

Stochastically sample from the $N_{\text{pos}}$ and $N_{\text{neg}}$ distributions, forming $N_{\text{net}}$ distributions from which one can calculate $S\sigma$ and $K\sigma^2$

Sampled Singles and IRV approach give the same results if former “oversampled” with weights …both/either can be called an “Independent Production” expectation

Other important “baselines” include

- **Poisson (Skellam)** – uncorrelated HG emission, calculable from $<N_{\text{pos}}>$ and $<N_{\text{neg}}>$ only
  

- **(N)BD** – sister functions to Poisson for which $\mu<\sigma^2$ (Neg. binomial) or $\mu>\sigma^2$ (binomial)
  
Corrected net-p $S\sigma$ and $K\sigma^2$ vs $\sqrt{s_{NN}}$ for 0-5% and 70-80% centrality

Independent production approaches reproduce the net-p moments products.
Efficiency-corrected net-p Kσ² vs centrality by \( \sqrt{s_{NN}} \)

IRV and sampled singles approaches (cyan) quantitatively reproduce the net-proton moments products at all beam energies and centralities…
Further exploring the moments products by deconstructing them…

The net-proton moments products can be understood using the p and pbar multiplicity distributions separately…

Intra-event correlations of $N_p$ and $N_{pbar}$ do not measurably affect the net-p moments products.

That is…

$$K\sigma^2_{(net-p)} = \frac{C_4(\text{net-p})}{C_2(\text{net-p})}$$

$$= \frac{[C_4(p)+C_4(pbar)]}{[C_2(p)+C_2(pbar)]}$$

Four quantities there.

Are the experimental values of $K\sigma^2_{(net-p)}$ driven by all four quantities equally? Or does one of these dominate?
Charge-separated uncorrected $K\sigma^2$ vs. centrality by $\sqrt{s_{NN}}$

$\sqrt{s_{NN}} < 27$ GeV ... $K\sigma^2$(net-$p$) = $K\sigma^2$(p)

$\sqrt{s_{NN}} \geq 39$ GeV ... $K\sigma^2$(pbar) > $K\sigma^2$(net-$p$) > $K\sigma^2$(p)

Uncorrected Au+Au
- • Measured $p$-$\bar{p}$
- △ Measured $p$
- ▲ Measured $\bar{p}$

Uncertainties are statistical only.
Uncorrected net-proton $C_2$ (variance) vs. centrality by $\sqrt{s_{NN}}$

- $C_2$ smoothly increases with $N_{part}$.
- $C_2$ changes with $\sqrt{s_{NN}}$.
- Uncertainties are statistical-only.

$C_2$ smoothly increases with $N_{part}$. ...changes with $\sqrt{s_{NN}}$. 
Uncorrected net-proton $C_4$ vs. centrality by $\sqrt{s}_{NN}$

- $p\bar{p} C_4$ vs $N_{part}$, $\sqrt{s}_{NN} = 7.7$ GeV
- $p\bar{p} C_4$ vs $N_{part}$, $\sqrt{s}_{NN} = 11.5$ GeV
- $p\bar{p} C_4$ vs $N_{part}$, $\sqrt{s}_{NN} = 19.6$ GeV
- $p\bar{p} C_4$ vs $N_{part}$, $\sqrt{s}_{NN} = 27.0$ GeV

Uncorrected Au+Au
- Measured $p\bar{p}$
- Measured $p$
- Measured $\bar{p}$

Uncertainties are statistical-only

- Proton $C_4$ sags for 0-5% @ 19&27...
- Pbar $C_4$ increases smoothly...
Net-Charge (results from AGS-RHIC Users Meeting 2013, D. McDonald)

Strong intra-event correlations!
Net-proton moments products

Independent random variable cumulant arithmetic and the “sampled singles” approaches reproduce the experimentally-measured net-proton moments products nearly exactly…

• Implies intra-event correlations of $N_p$ and $N_{p\bar{p}}$ do not have a measurable effect on the measured net-p moments products.

• Agreement is almost as good if one simply ignores the antiprotons.

• The dip with respect to Poisson near $\sim 19.6$ GeV is driven by the proton $C_4$ values…
  …proton $C_2$ smoothly increases with centrality and beam energy

\[ K\sigma^2(\text{net-p}) = \frac{C_4(\text{net-p})}{C_2(\text{net-p})} = \frac{[C_4(p)+C_4(p\bar{p})]}{[C_2(p)+C_2(p\bar{p})]} \]

Net-charge moments products

• Net-charge moments products deviate slightly from all baselines in general.

• Independent production approaches deviate strongly from the experimental data for $\sim$central collisions in the 62.4 and 200 GeV data sets.
Comparison of (N)BD to uncorrected net-proton $C_4$ vs. centrality by $\sqrt{s_{NN}}$

Uncorrected Au+Au
- Measured $p-\bar{p}$
- (N)BD $p-\bar{p}$
- Measured $p$
- (N)BD $p$
- Measured $\bar{p}$
- (N)BD $\bar{p}$

uncertainties are statistical-only

(N)BD $C_4 \neq$ data $C4$ for 0-5% @ 19 GeV…
On the apparent net-p dip near 19.6 GeV

J. Nagle, last talk at QM2012

Kurtosis < Poisson for $\sqrt{s_{NN}}$ just above CP?

M.A. Stephanov, Phys. Rev. Lett. 107, 052301 (2011)
On the apparent net-p dip near 19.6 GeV

J. Nagle, last talk at QM2012

what the NLSM would *actually* expect for a CP at $\sqrt{s_{NN}} \sim 15$ GeV
(15 GeV data on the way in upcoming Run 14!)

Functional form describes the (particle identified) multiplicity distributions ranging from NA22 & UA5 to PHENIX

Inputs:
mean (µ) & variance (σ²)

Then, the values of C_k, Sσ, & Kσ² are predicted.

µ<σ²  ...NBD
µ=σ²  ...Poisson
µ>σ²  ...BD

If the mean is equal to the variance, a Poisson Distribution is used. The BD baseline for the moments products Sσ and Kσ² uses the parameter p, defined as p=1-σ²/µ, where µ is the mean and σ² is the variance. Then, the BD baseline for Sσ is given by

\[(S\sigma)^{BD} = 1 - 2p (\mu > \sigma^2), \quad (2.19)\]

and the BD calculation of the moments product Kσ² is

\[(K\sigma^2)^{BD} = 1 - 6p + 6p^2 (\mu > \sigma^2). \quad (2.20)\]

For the NBD baselines of the moments products Sσ and Kσ² the parameter p is defined as p=µ/σ². Then, the NBD baseline for Sσ is given by

\[(S\sigma)^{NBD} = (2 - p)/p (\mu < \sigma^2), \quad (2.21)\]

and the NBD calculation of the moments product Kσ² is

\[(K\sigma^2)^{NBD} = (6 - 6p + p^2)/p^2 (\mu < \sigma^2). \quad (2.22)\]
The only input is the 2D distributions of $N_{pos}$ vs. centrality and $N_{neg}$ vs. centrality, where $pos = p, K^+, q^+$ & $neg = \bar{p}, K^-, q^-$

With $N_{net}$ and $N_{tot}$ vs. centrality I can also independently produce the experimental results, with delta theorem error bars, efficiency corrections, etc…

Filled at exactly the same spot in the analysis codes where the deviates are saved

*i.e.* TH2Ds include the same track cuts, PID, and run&evt QA as the local analysis…
In every slice of \( \text{rmXcorr} \), sample one value of \( N_{\text{pos}} \) and one value of \( N_{\text{neg}} \), \( N_{\text{evt}} \) times.

In each sample at a given \( \text{rmXcorr} \), one then has a value for \( N_{\text{pos}} \) and \( N_{\text{neg}} \).

Then form \( N_{\text{net}} = N_{\text{pos}} - N_{\text{neg}} \) and \( N_{\text{tot}} = N_{\text{pos}} + N_{\text{neg}} \).

Fill similar 2D plots of \( N_{\text{net}} \) and \( N_{\text{tot}} \) vs. centrality.

And then extract the moments (products) and do the CBW corrections as usual…

Destroys all intra-event correlations between \( N_{\text{pos}} \) and \( N_{\text{neg}} \), reproduces singles distributions, & has the same statistical certainty as the data by construction…

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cf. G. Torrieri et al.,