Highlights of the Multiplicity Moments Analyses in the Bulk Correlations PWG

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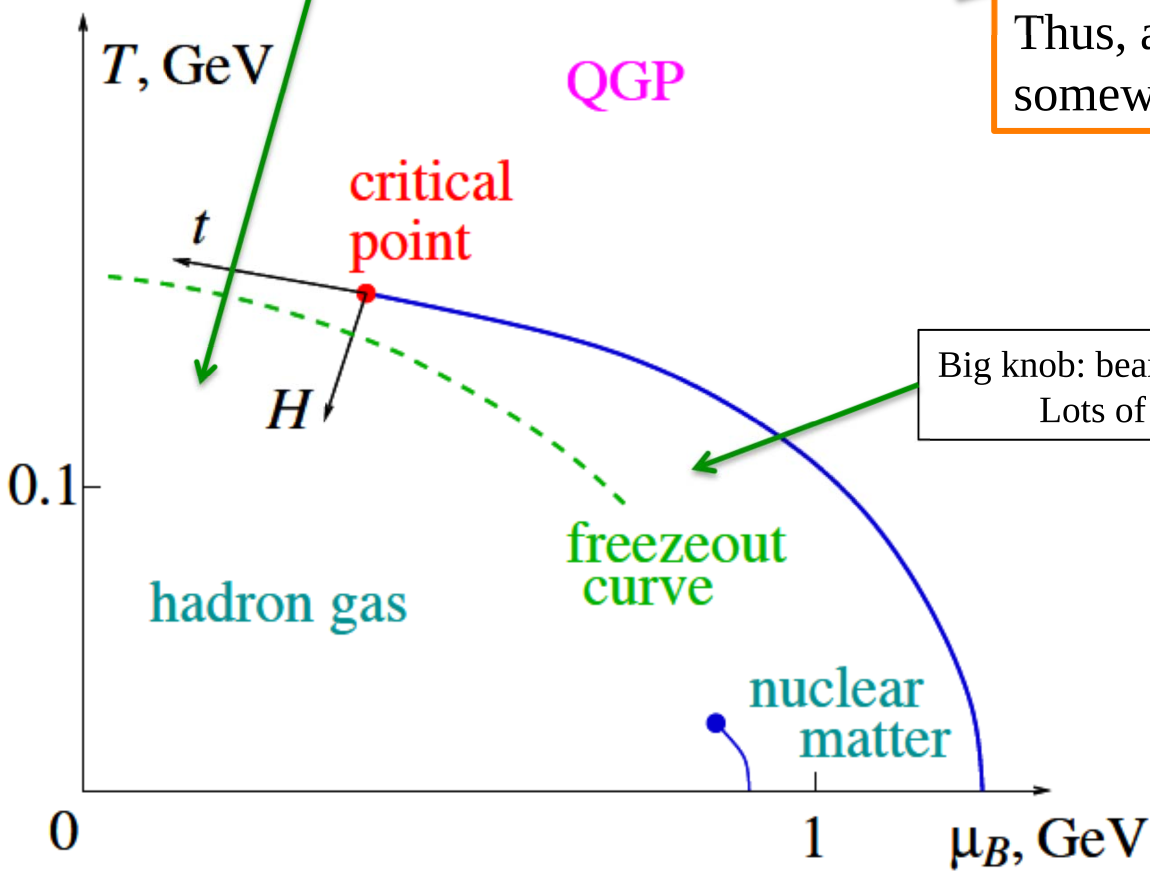
Big knob: beam energy is large…
 2nd order in the Chiral Limit ($m_q = 0$)
 Cross-over with $m_q \neq 0$

Thus, a (2nd order) CP must exist somewhere in between these extremes…..



Big knob: beam energy is small….
 Lots of evidence for 1st order…

Finding the CP in the theory (lattice, σ-model) is hard…
- finite lattice
- fermion sign problem
- quark masses, s-quark quenching

So, find the CP experimentally!

1007.2613

Zooming in on the Phase Diagram

So how do we find such a Critical Point?

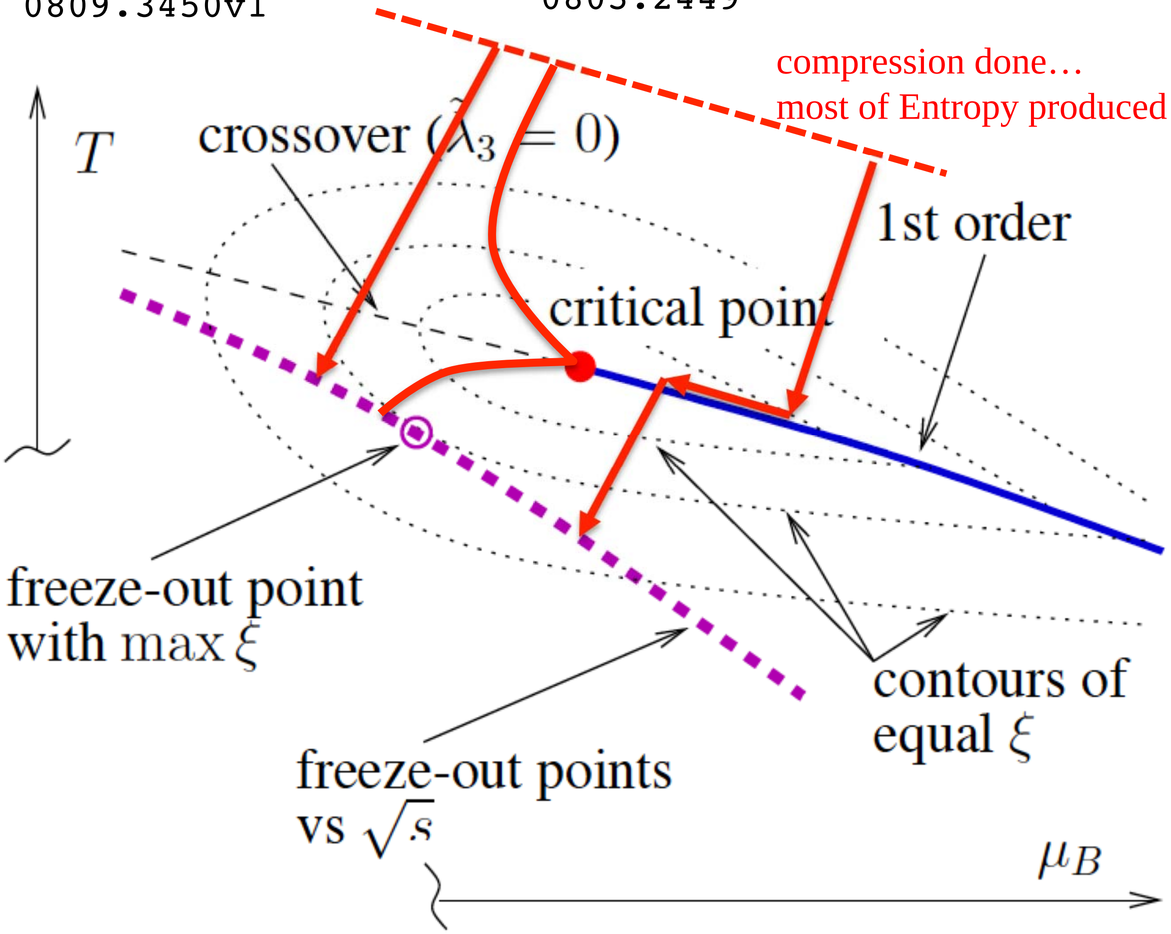
Make the assumption that it’s going to have the same basic features of other CPs!

divergence of the susceptibilities, χ… *e.g.* magnetism transitions 0801.4256v2

divergence of the correlation lengths, ξ… *e.g.* critical opalescence at water’s CP

hep-ph/0402115
0809.3450v1

0803.2449



Life is a little harder for us…

- finite system size
- finite system lifetime
- critical “slowing down”

hep-ph/0912274v2

…ξ simply cannot reach ∞…

- ξ_{max} ~ 2-3 fm
- $\xi_{\text{“natural”}}$ ~ 0.5 fm

…How close is FO curve to CP?

concentrate on observables that are extremely sensitive to the value of ξ…

Moments of particle multiplicities related to conserved qtys:

B, Q, S

Sensitivity of cumulants to the Critical Point from the theory

Multiplicity cumulants related to conserved qty's Q,B,S can be calculated in phenomenological models and on the lattice…

$$\chi_n^B = -\frac{1}{3^n}\frac{\partial^n f/T^4}{\partial \hat{\mu}_q^n}$$

m_1	$= S\sigma$	$\sim \chi^{(3)}T / [\chi^{(2)}T^2]$	$\sim \xi^{7/2}$	Skewness * Standard Deviation
m_2	$= K\sigma^2$	$\sim \chi^{(4)} / [\chi^{(2)}T^2]$	$\sim \xi^{5}$	Kurtosis * Variance
m_3	$= K\sigma/S$	$\sim \chi^{(4)}T / [\chi^{(2)}/T]$	$\sim \xi^{5/2}$	Kurtosis * Standard Deviation / Skewness

In the "σ model," the cumulants of the occupation numbers (integral=multiplicity) are directly related to ξ… and, likewise, the higher the moment, the higher the dependence of the moment on ξ….

$$\kappa_2 = \langle\sigma_0^2\rangle = \frac{T}{V}\xi^2\,; \qquad \kappa_3 = \langle\sigma_0^3\rangle = \frac{2\lambda_3 T}{V}\xi^6\,;$$

$$\kappa_4 = \langle\sigma_0^4\rangle_c \equiv \langle\sigma_0^4\rangle - \langle\sigma_0^2\rangle^2 = \frac{6T}{V}[2(\lambda_3\xi)^2 - \lambda_4]\,\xi^8$$

0809.3450v1

$$\delta x \equiv x - \langle x\rangle$$

$$\kappa_{2x} \equiv \langle\langle x^2\rangle\rangle \equiv \langle(\delta x)^2\rangle$$

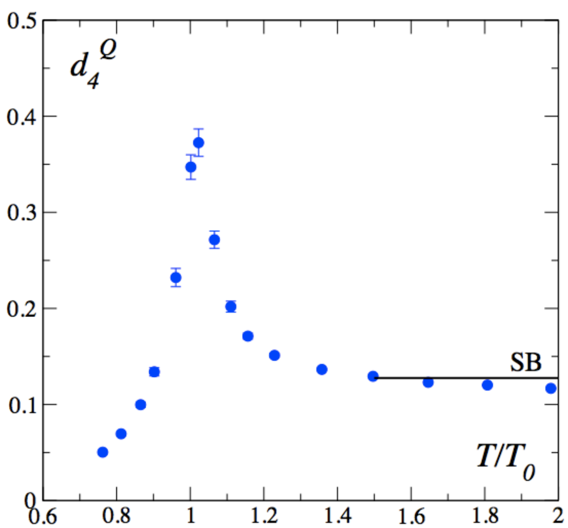
$$\kappa_{3x} \equiv \langle\langle x^3\rangle\rangle \equiv \langle(\delta x)^3\rangle$$

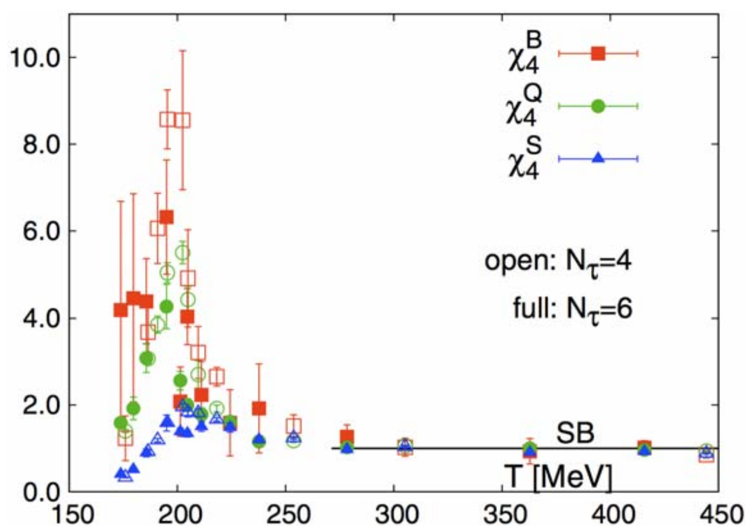
$$\kappa_{4x} \equiv \langle\langle x^4\rangle\rangle \equiv \langle(\delta x)^4\rangle - 3\langle(\delta x)^2\rangle^2$$

$$\omega_{ip} \equiv \frac{\kappa_{ip}}{\langle N_p\rangle}$$

$$\text{skewness} = \frac{\kappa_3}{\kappa_2^{3/2}},\ \text{kurtosis} = \frac{\kappa_4}{\kappa_2^2}$$

0509051 0610116 PhysRevD.79.074505



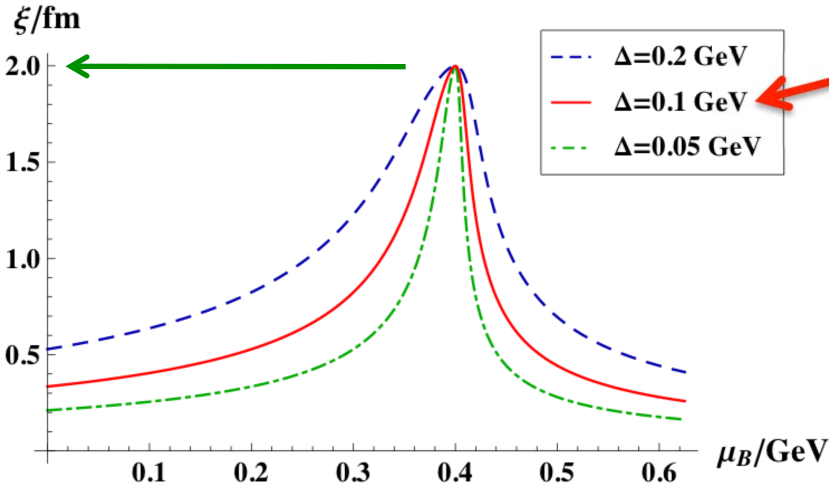


We can attack this, via "higher moments" of proxy observables:

- B: net-protons
- Q: net-charge
- S: net-strangeness

Sensitivity Continued

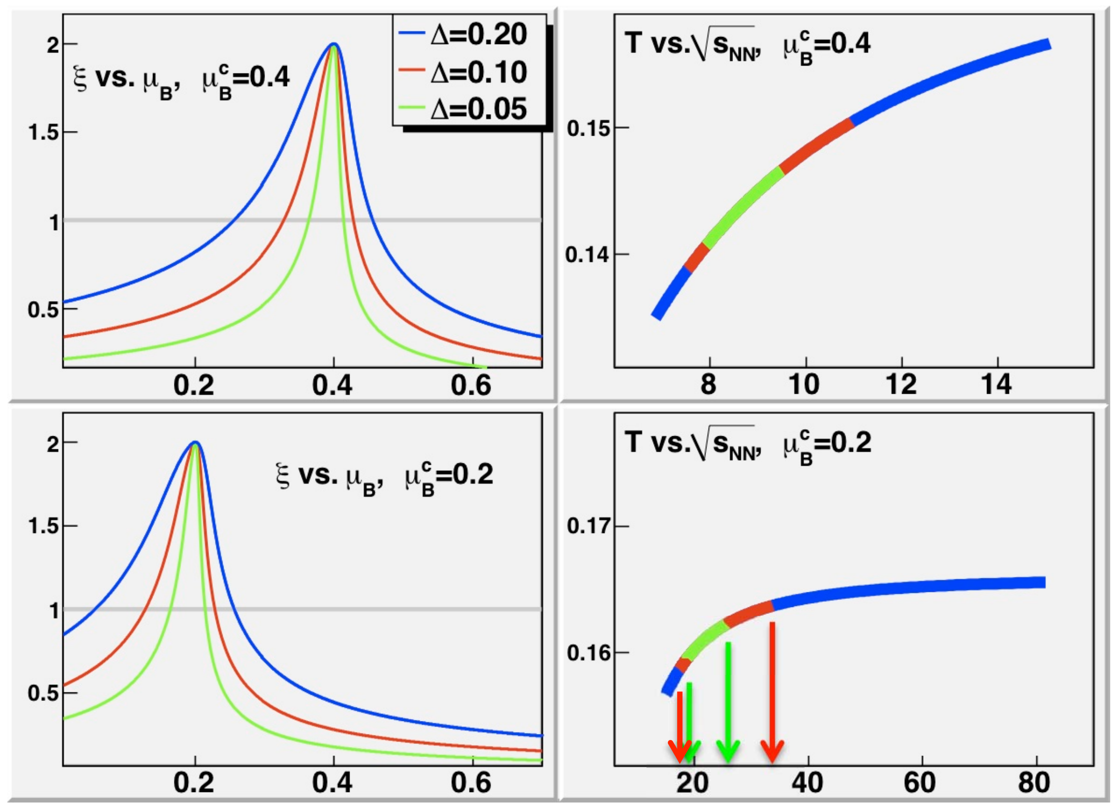
An ansatz reproducing the expected trends: 1006.4636v2



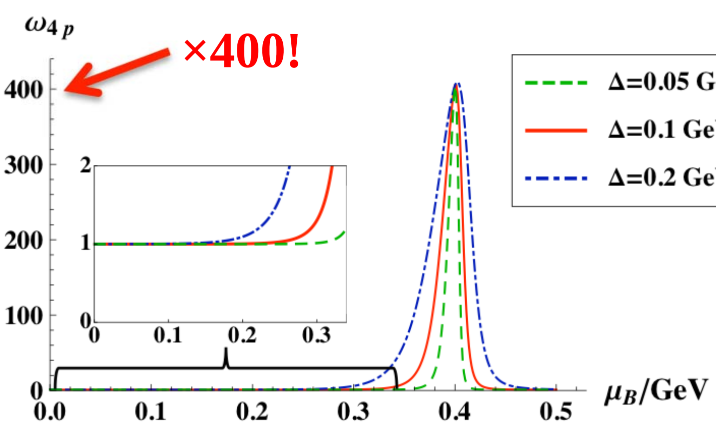
lattice suggests this value… PRD 78,114503 (2008)

this ansatz in "experimental units"

convert μ_B into $\sqrt{s_{NN}}$ and T for ξ>1fm
using ($\sqrt{s_{NN}}$,μ_B,T)parameterizations in 0511094



results in huge increases in the
4th proton cumulant and others….



i.e. perhaps μ_B ($\sqrt{s_{NN}}$) doesn't have to be right on top of the CP to see the effects on the moments….

Experimental Approach

Seems easy, right? …All we have to do is

Constrain the "big knob" ($\sqrt{s_{NN}}$ *i.e.* μ_B) and "fine knobs" (b, y)…

& then measure the moments μ, σ, S, K or norm. cumulants of our particle proxies…

& then form various products and ratios of these…

& then plot the observable vs. the big knob…

Golden signal is a "non-monotonic behavior" of high moments or cumulants vs the knobs.

But not so fast….

- Are our proxies a reasonable approximation for the conserved qty (Q,B,S)?

  this must be checked. Net-protons are reasonable for B… What about the others?

  PRL 105, 022302 (2010)

- All "non-thermal" sources of fluctuations must be removed!

physical:	Volume fluctuations (control via centrality cuts)
	Jets, weak decays, *etc.*
experimental:	Time dependence in run (bad RDOs, missing TOF sectors, *etc.*)
	Time dependence in fills (backgrounds) and, esp. at low $\sqrt{s_{NN}}$, dependence on tuning
	Effects of event & track quality cuts, PID cuts, dE/dx vs dE/dx+TOF, *etc*…
	Autocorrelations between centrality cuts and moments
	Dependence of moments values on width of centrality bins ("CBWE")…

Only one choice: Look at all particle groups, calculate every qty (moments, cumulants, ratios), and then try to figure out what, if anything, is CP physics and what is experimental...

Bulkcorr "Moments" Analyses

Many independent parallel analyses are underway – Lots of interest in this direction!

Xiaofeng Luo	net-p $S\sigma$ and $K\sigma^2$
Nihar Sahoo	net-h $S\sigma$ and $K\sigma^2$
Gary Westfall	net-p and net-K $S\sigma$ and $K\sigma^2$
Lizhu Chen	net-p & net-K, NIC, C_6/C_2
Daniel McDonald	8 groups (4 net and 4 total), NIC, $S\sigma$, $K\sigma^2$
Zhiming Li & Yuanfeng Wu	net-p $S\sigma$, $K\sigma^2$, S_{dyn}, ν_{dyn}

Note also an impressive set of results from ν_{dyn} analyses of various particle ratios

Terry Tarnowsky, Gary Westfall, Jian Tian

Pretty much the same fluctuation physics cast into a slightly different language....

similar baseline subtraction towards "dynamic" fluctuations to the moments (Skellam) exists…

1011.0712

Many (potentially important) differences between these analyses….

- centrality selection ("refmult" standard, vs "refmult2") & Autocorrelations…
- event & track cuts…
- |y| *vs.* |η| windows…
- calculated error bars *vs.* 5-subgroup errors…

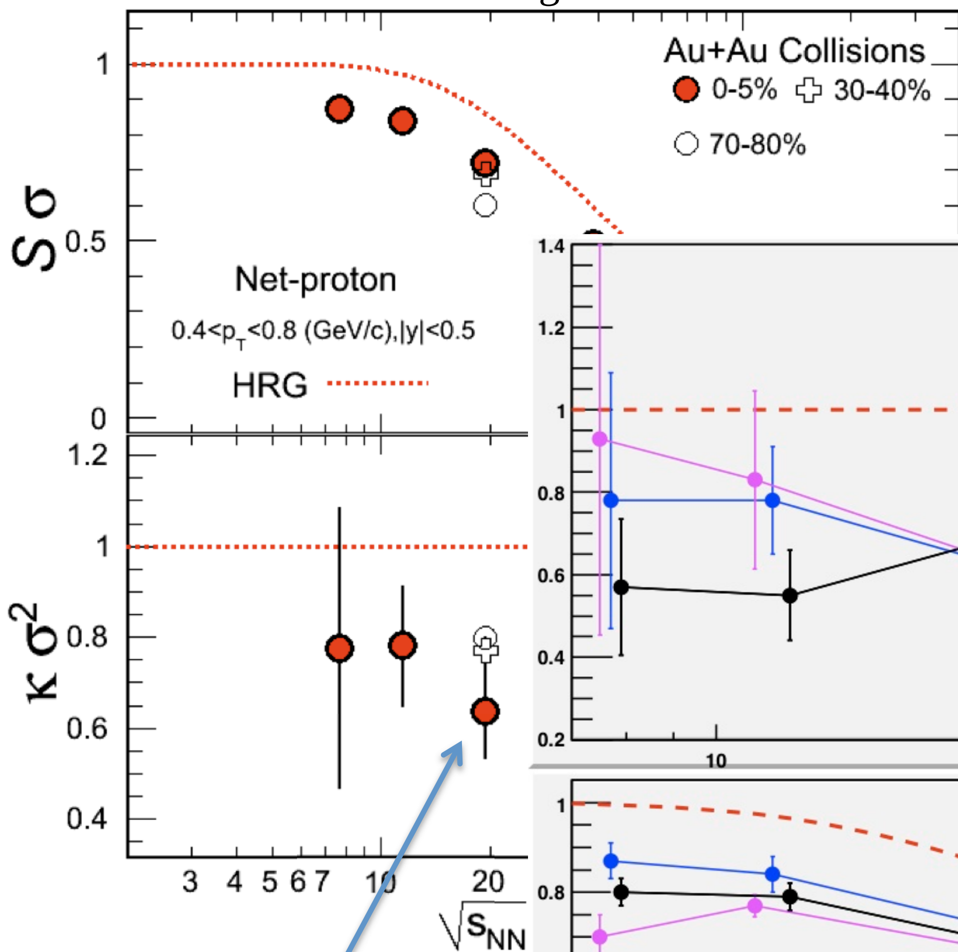
details….

http://wjllope.rice.edu/fluct/protected/MomentsCuts_20110629.pdf

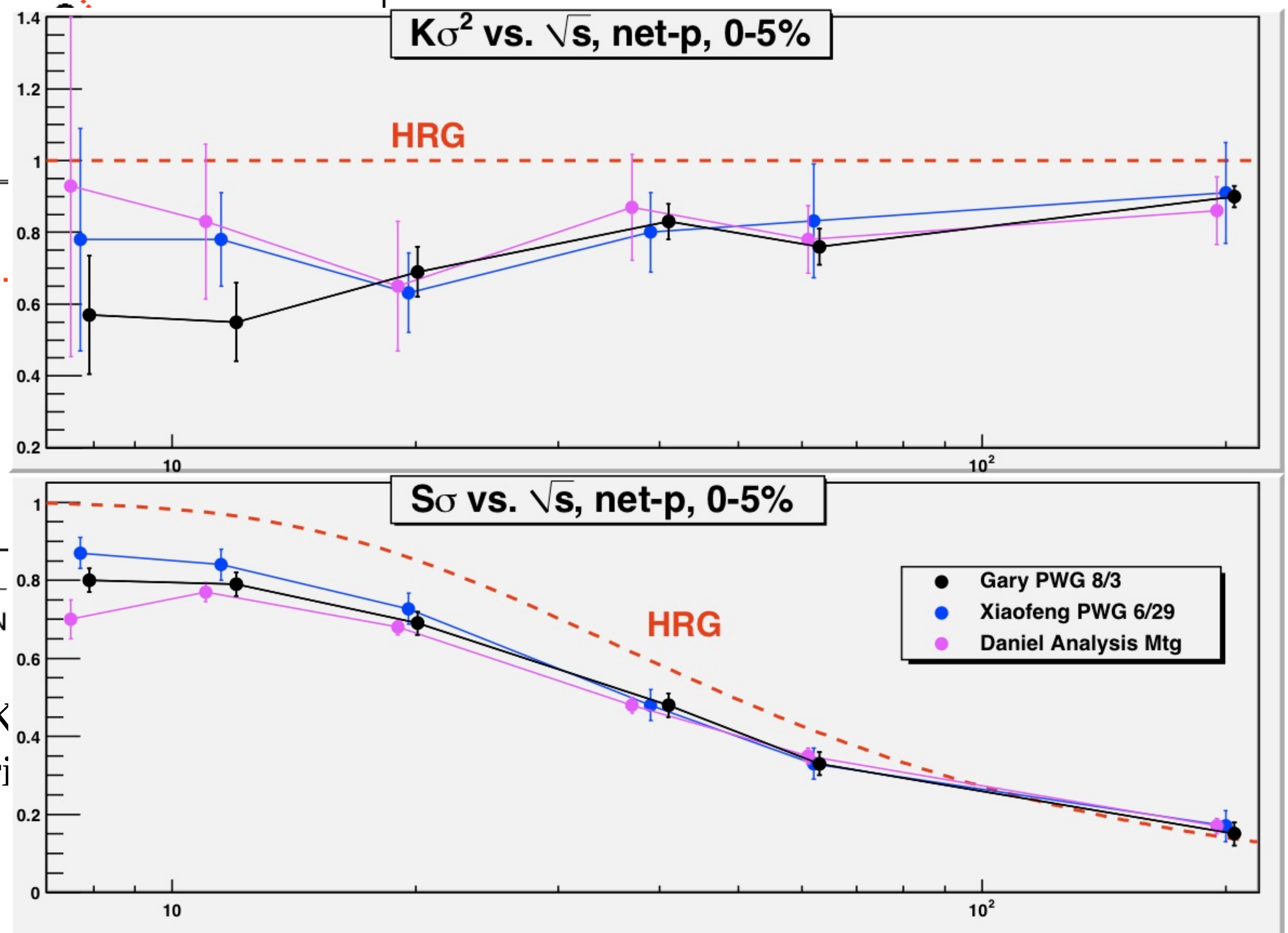
Lots for us to understand quantitatively still, but let's move on to some plots….

net-protons

Xiaofeng Luo



plot by Daniel McDonald

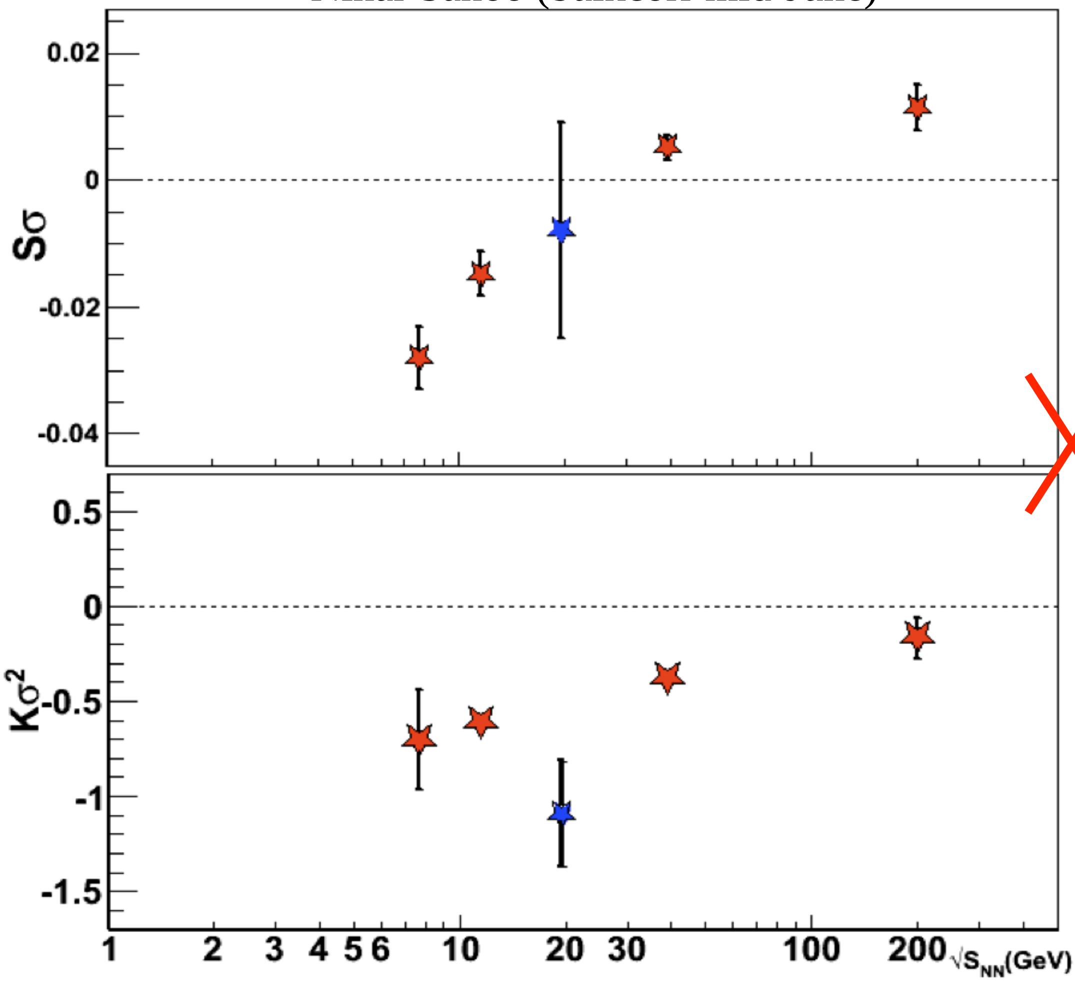


~1σ minimum of K

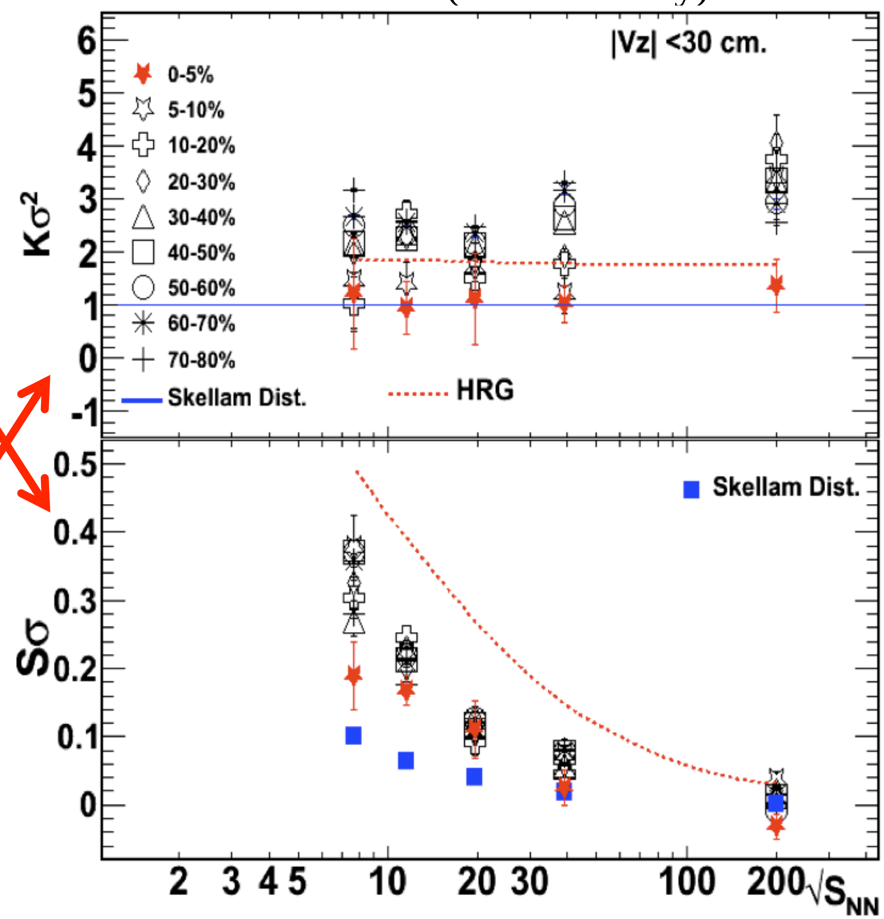
Peripheral events ri

net-charge

Nihar Sahoo (bulkcorr mid-June)



Nihar Sahoo (more recently)



$S\sigma<0$ then $S>0$ w/ $\sqrt{s_{NN}}$, $K\sigma^2$ negative at all $\sqrt{s_{NN}}$…
~1.5σ minimum of $K\sigma^2$ at 19.6 GeV…

$S\sigma$ & $K\sigma^2$ now positive at all $\sqrt{s_{NN}}$…
no minima anymore…

centrality selection changed ("refmult2")

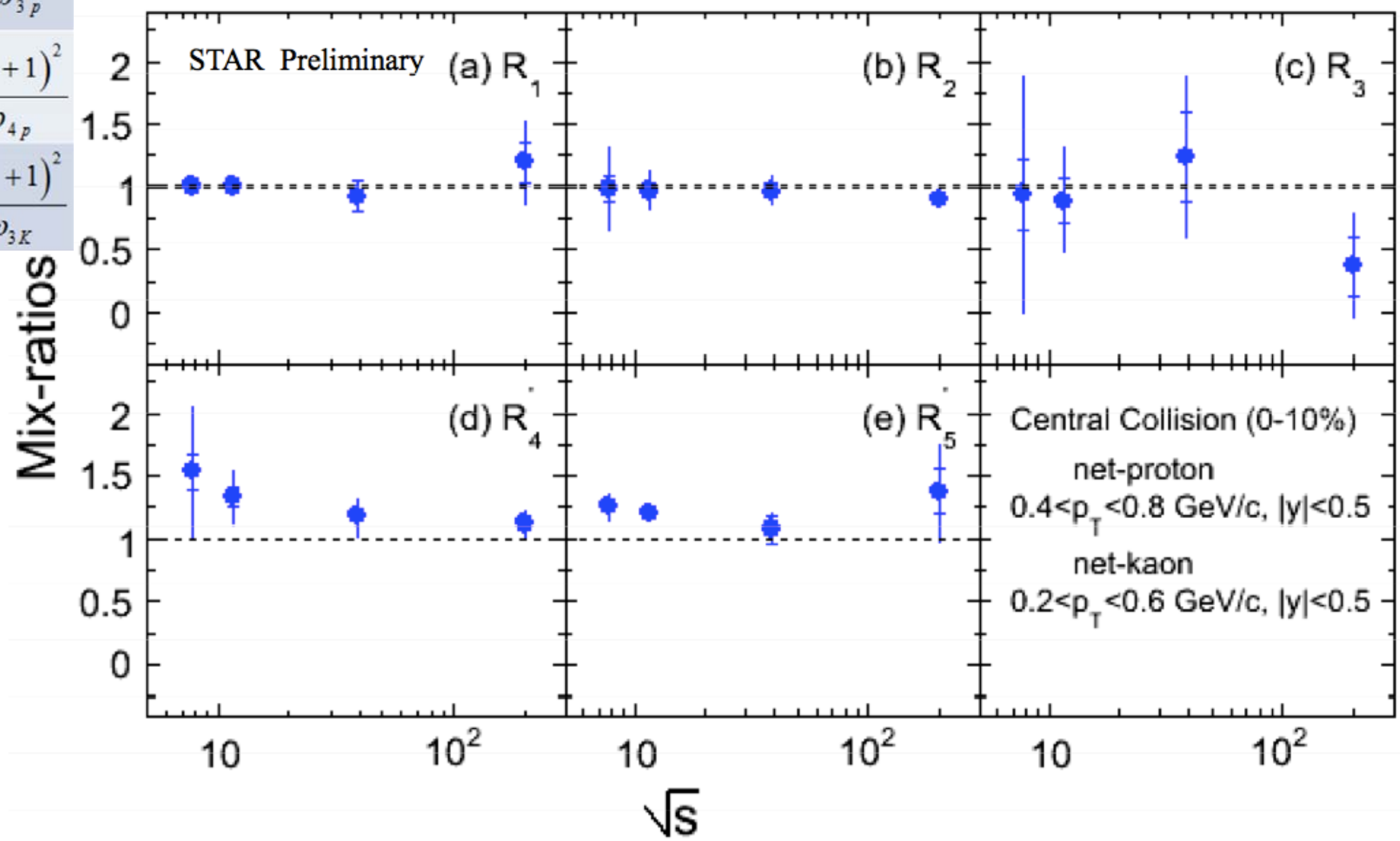
“mixed ratios”

Original ratios	Reduced ratios
$R_1 = \frac{\kappa_{3p}\kappa_{2K}^{3/2}}{\kappa_{3K}\kappa_{2p}^{3/2}}$	$R_1 = \frac{\omega_{3p}\omega_{2K}^{3/2}}{\omega_{3K}\omega_{2p}^{3/2}}$
$R_2 = \frac{\kappa_{4p}\kappa_{2K}^{2}}{\kappa_{4K}\kappa_{2p}^{2}}$	$R_2 = \frac{\omega_{4p}\omega_{2K}^{2}}{\omega_{4K}\omega_{2p}^{2}}$
$R_3 = \frac{\kappa_{4p}^{3}\kappa_{3K}^{4}}{\kappa_{4K}^{3}\kappa_{3p}^{4}}$	$R_3 = \frac{\omega_{4p}^{3}\omega_{3K}^{4}}{\omega_{4K}^{3}\omega_{3p}^{4}}$
$R_4 = \frac{\kappa_{2p2K}^{2}}{\kappa_{4K}\kappa_{4p}}$	$R_4^{'} = \frac{(\omega_{2p2K}+1)^2}{\omega_{4K}\omega_{4p}}$
$R_5 = \frac{\kappa_{2p1K}^{3}}{\kappa_{3p}^{2}\kappa_{3K}}$	$R_5^{'} = \frac{(\omega_{2p1K}+1)^2}{\omega_{3p}^{2}\omega_{3K}}$

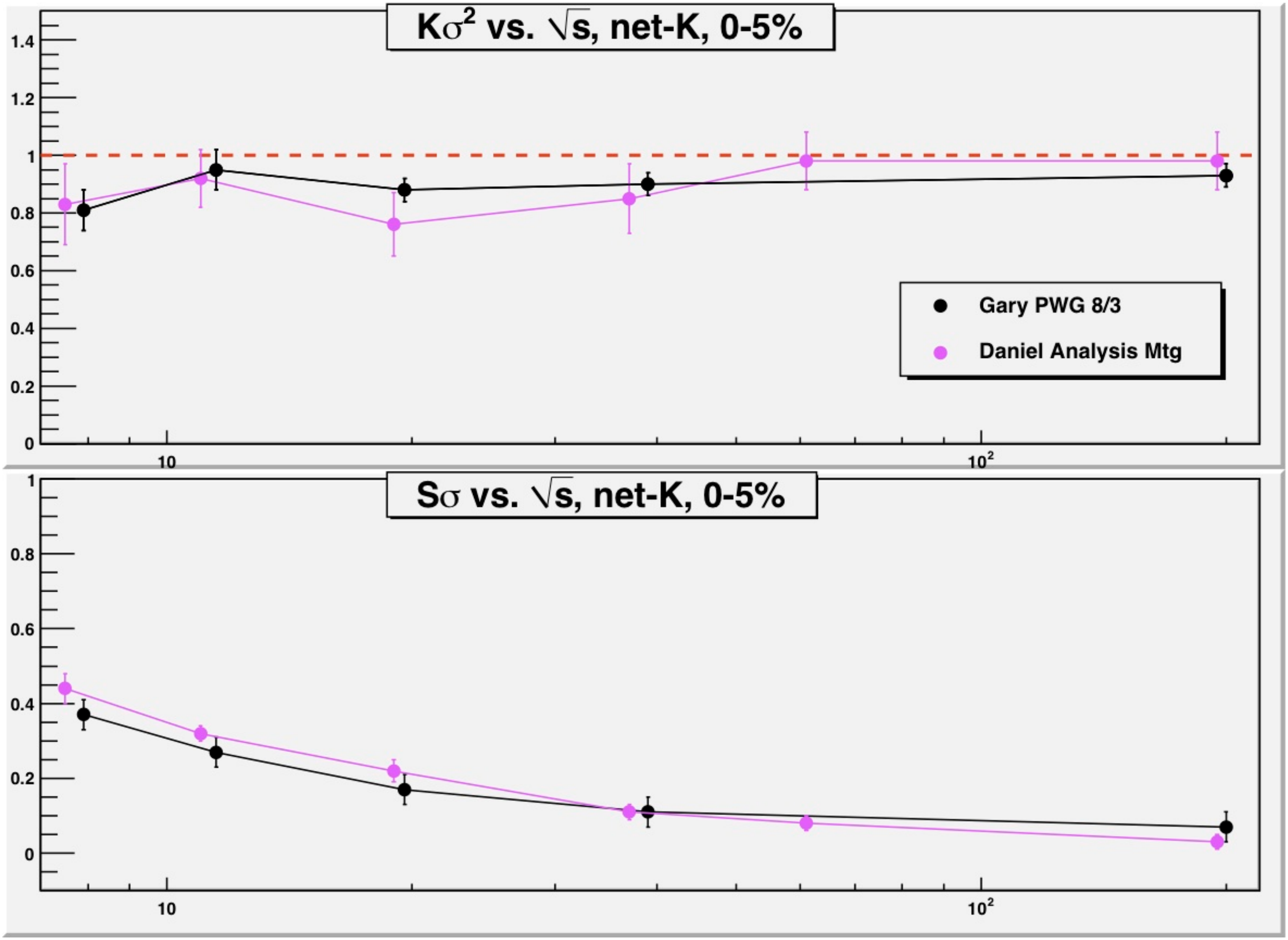
Lizhu Chen

These ratios should be =1 even if non-monotonic behavior seen in other cumulants…. 1006.4636

A good test of analysis systematics…



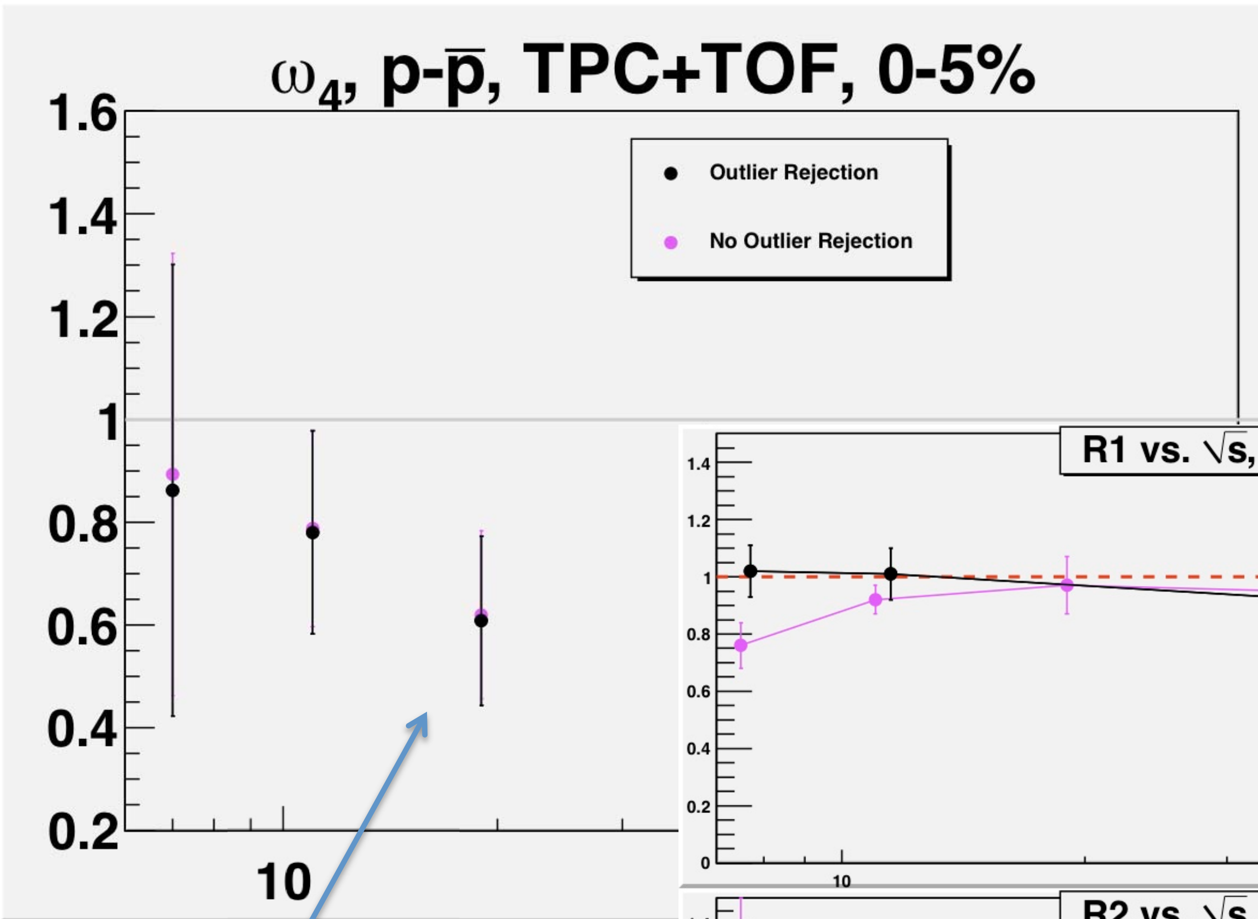
net Kaons

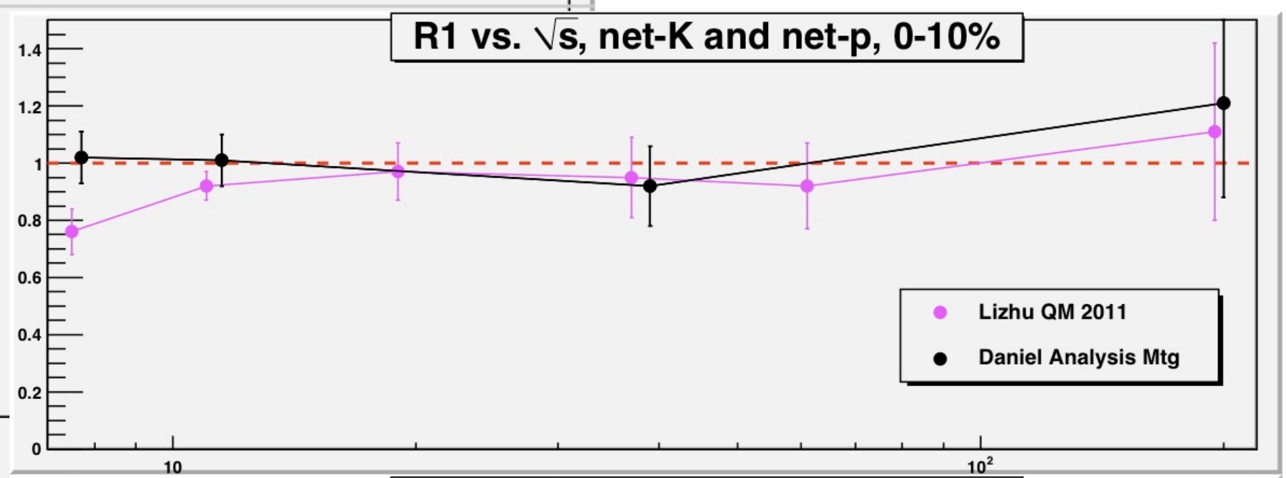

Kσ² vs. √s, net-K, 0-5%
1.4
1.2
1
0.8
0.6
0.4
0.2
0
10
10²
Gary PWG 8/3
Daniel Analysis Mtg
Sσ vs. √s, net-K, 0-5%
1
0.8
0.6
0.4
0.2
0
10
10²

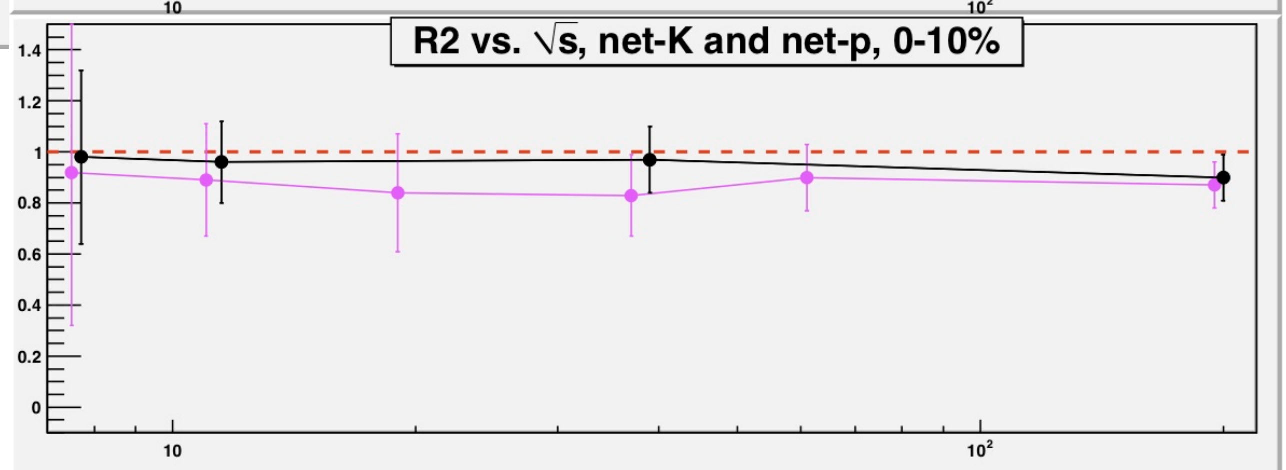
Normalized Intensive Cumulants

Daniel McDonald

careful outlier rejection now implemented







possible shallow minimum at ~19.6 (0.5σ)

Summary

moments & cumulants suggested to be extremely sensitive to proximity to CP
possible shallow minimum in net-p near 19.6 GeV, but not reproduced by others…
possible shallow minimum in net-h when using refmult, but disappears for refmult2…
possible shallow minima in other NICs…

so far, we see no excursion from Poisson-like trends greater than ~1σ…

either Δ is very narrow, or we still have a ways to go to understand the systematics…
- |η| windows vs |y| windows…
- centrality selection and Autocorrelations…
  - refmult vs refmult2
  - more forward detectors? BEMC energy (mainly π^0)?
- event and track quality cuts… PID methods…. Excluded run criteria…
- subgroup vs calculated error bars…

And, the higher the moment, the larger the error bars…. (*cf.* Evan Sangeline and Jim Draper)

Tomorrow morning:

Bulk Correlations PWG Parallel Session: Moments Workshop (185 Physics : 08:00 - 12:20)

08:00 **Higher Moments of Net-Charge Distribution - Nihar Sahoo (VECC)**
08:30 **[SQM Preview] Search for the Critical Point with Higher Moments of Net-proton Mulitplicity - Xiaofeng Luo (USTC)**
09:00 **Moments Analysis - Daniel McDonald (Rice)**
09:30 **Coffee Break (00:30)**
10:00 **Net Particle Higher Moments - Gary Westfall (Michigan State)**
10:30 **Mix Ratios - Lizhu Chen (IPP Wuhan)**