Highlights of the Multiplicity Moments Analyses in the Bulk Correlations PWG

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Big knob: beam energy is large…
2nd order in the Chiral Limit ($m_q = 0$)
Cross-over with $m_q \neq 0$

Thus, a (2\textsuperscript{nd} order) CP must exist somewhere in between these extremes…..

Finding the CP in the theory (lattice, $\sigma$-model) is hard…
finite lattice
fermion sign problem
quark masses, s-quark quenching
So, find the CP experimentally!

1007.2613
So how do we find such a Critical Point?

Make the assumption that it’s going to have the same basic features of other CPs!

- divergence of the susceptibilities, $\chi$... e.g. magnetism transitions
- divergence of the correlation lengths, $\xi$... e.g. critical opalescence at water’s CP

Life is a little harder for us...

- finite system size
- finite system lifetime
- critical “slowing down”

...$\xi$ simply cannot reach $\infty$...

$\xi_{\text{max}} \sim 2-3$ fm
$\xi_{\text{natural}} \sim 0.5$ fm

...How close is FO curve to CP?

concentrate on observables that are extremely sensitive to the value of $\xi$...

Moments of particle multiplicities related to conserved qtys:

B, Q, S
Sensitivity of cumulants to the Critical Point from the theory

Multiplicity cumulants related to conserved qty’s Q,B,S can be calculated in phenomenological models and on the lattice…

\[ m_1 = S \sigma \sim \frac{\chi^{(3)}T}{[\chi^{(2)}T^2]} \sim \xi^{7/2} \]  
Skewness * Standard Deviation

\[ m_2 = K \sigma^2 \sim \frac{\chi^{(4)}}{[\chi^{(2)}T^2]} \sim \xi^5 \]  
Kurtosis * Variance

\[ m_3 = K \sigma / S \sim \frac{\chi^{(4)}T}{[\chi^{(2)}T]} \sim \xi^{5/2} \]  
Kurtosis * Standard Deviation / Skewness

In the “σ model,” the cumulants of the occupation numbers (integral=multiplicity) are directly related to \( \xi \)… and, likewise, the higher the moment, the higher the dependence of the moment on \( \xi \)…

\[ \kappa_2 = \langle \sigma_0^2 \rangle = \frac{T}{V} \xi^2; \quad \kappa_3 = \langle \sigma_0^3 \rangle = \frac{2\lambda_3 T}{V} \xi^6; \]

\[ \kappa_4 = \langle \sigma_0^4 \rangle_c \equiv \langle \sigma_0^4 \rangle - \langle \sigma_0^2 \rangle^2 = \frac{6T}{V} [2(\lambda_3 \xi)^2 - \lambda_4] \xi^8 \]

\[ \delta x \equiv x - \langle x \rangle \]

\[ \kappa_{2x} \equiv \langle (\delta x)^2 \rangle \equiv \langle (\delta x)^2 \rangle \]

\[ \kappa_{3x} \equiv \langle (\delta x)^3 \rangle \equiv \langle (\delta x)^3 \rangle \]

\[ \kappa_{4x} \equiv \langle (\delta x)^4 \rangle \equiv \langle (\delta x)^4 \rangle - 3 \langle (\delta x)^2 \rangle^2 \]

\[ \omega_{ip} \equiv \frac{\kappa_{ip}}{\langle N_p \rangle} \]

skewness = \( \frac{\kappa_3}{\kappa_2^{3/2}} \), kurtosis = \( \frac{\kappa_4}{\kappa_2^2} \)

We can attack this, via “higher moments” of proxy observables:

- B: net-protons
- Q: net-charge
- S: net-strangeness
An ansatz reproducing the expected trends: $1006.4636v^2$

results in huge increases in the 4th proton cumulant and others...  

$i.e.$ perhaps $\mu_B (\sqrt{s_{NN}})$ doesn’t have to be right on top of the CP to see the effects on the moments...
Seems easy, right? ...All we have to do is
Constrain the “big knob” ($\sqrt{s_{NN}}$ i.e. $\mu_B$) and “fine knobs” (b, y)...
& then measure the moments $\mu$, $\sigma$, S, K or norm. cumulants of our particle proxies...
& then form various products and ratios of these...
& then plot the observable vs. the big knob...

Golden signal is a “non-monotonic behavior” of high moments or cumulants vs the knobs.

But not so fast….

• Are our proxies a reasonable approximation for the conserved qty (Q,B,S)?
  this must be checked. Net-protons are reasonable for B… What about the others?

• All “non-thermal” sources of fluctuations must be removed!
  physical: Volume fluctuations (control via centrality cuts)
  Jets, weak decays, etc.
  experimental: Time dependence in run (bad RDOs, missing TOF sectors, etc.)
  Time dependence in fills (backgrounds) and, esp. at low $\sqrt{s_{NN}}$, dependence on tuning
  Effects of event & track quality cuts, PID cuts, dE/dx vs dE/dx+TOF, etc…
  Autocorrelations between centrality cuts and moments
  Dependence of moments values on width of centrality bins (“CBWE”)…

Only one choice: Look at all particle groups, calculate every qty (moments, cumulants, ratios),
and then try to figure out what, if anything, is CP physics and what is experimental...
Many independent parallel analyses are underway – Lots of interest in this direction!

Xiaofeng Luo net-p $S\sigma$ and $K\sigma^2$
Nihar Sahoo net-h $S\sigma$ and $K\sigma^2$
Gary Westfall net-p and net-K $S\sigma$ and $K\sigma^2$
Lizhu Chen net-p & net-K, NIC, $C_6/C_2$
Daniel McDonald 8 groups (4 net and 4 total), NIC, $S\sigma$, $K\sigma^2$
Zhiming Li & Yuanfeng Wu net-p $S\sigma$, $K\sigma^2$, $S_{dy}$, $\nu_{dy}$

Note also an impressive set of results from $\nu_{dy}$ analyses of various particle ratios
Terry Tarnowsky, Gary Westfall, Jian Tian
Pretty much the same fluctuation physics cast into a slightly different language....
similar baseline subtraction towards “dynamic” fluctuations to the moments (Skellam) exists…

Many (potentially important) differences between these analyses....
centrality selection (“refmult” standard, vs “refmult2”) & Autocorrelations…
event & track cuts…
$|y|$ vs. $|\eta|$ windows…
calculated error bars vs. 5-subgroup errors…
details....
http://wjllope.rice.edu/fluct/protected/MomentsCuts_20110629.pdf

Lots for us to understand quantitatively still, but let’s move on to some plots....
Bulkcorr Moments Highlights

Xiaofeng Luo

Net-protons

$0.4 < p_T < 0.8$ (GeV/c), $|y| < 0.5$

HRG

$\sigma$ minimum of $K$

Peripheral events rise to trend...

~1σ minimum of $K$

Plot by Daniel McDonald
S\sigma < 0 \text{ then } S > 0 \text{ w/ } \sqrt{s_{NN}}, \text{K}\sigma^2 \text{ negative at all } \sqrt{s_{NN}} \ldots \text{~1.5}\sigma \text{ minimum of } \text{K}\sigma^2 \text{ at 19.6 GeV} \ldots

S\sigma \text{ & K}\sigma^2 \text{ now positive at all } \sqrt{s_{NN}} \ldots \text{no minima anymore} \ldots

\text{centrality selection changed ("refmult2")}
Lizhu Chen

These ratios should be =1 even if non-monotonic behavior seen in other cumulants.

A good test of analysis systematics…
Bulkcorr Moments Highlights

$K_{\sigma}^2$ vs. $\sqrt{s}$, net-K, 0-5%

$S_{\sigma}$ vs. $\sqrt{s}$, net-K, 0-5%

[Graph showing data points and trends]

- Gary PWG 8/3
- Daniel Analysis Mtg
Possible shallow minimum at ~19.6 (0.5σ)

careful outlier rejection now implemented
moments & cumulants suggested to be extremely sensitive to proximity to CP
possible shallow minimum in net-p near 19.6 GeV, but not reproduced by others…
possible shallow minimum in net-h when using refmult, but disappears for refmult2…
possible shallow minima in other NICs…

so far, we see no excursion from Poisson-like trends greater than ~1σ…

either $\Delta$ is very narrow, or we still have a ways to go to understand the systematics…

$|\eta|$ windows vs $|y|$ windows…
centrality selection and Autocorrelations…
  refmult vs refmult2
  more forward detectors? BEMC energy (mainly $\pi^0$)?
event and track quality cuts… PID methods… Excluded run criteria…
subgroup vs calculated error bars…

And, the higher the moment, the larger the error bars…. (cf. Evan Sangeline and Jim Draper)

Tomorrow morning:

**Bulk Correlations PWG Parallel Session: Moments Workshop (185 Physics : 08:00 - 12:20)**

08:00 [Higher Moments of Net-Charge Distribution - Nihar Sahoo (VECC)]
08:30 [SQM Preview] Search for the Critical Point with Higher Moments of Net-proton Multiplicity - Xiaofeng Luo (USTC)
09:00 Moments Analysis - Daniel McDonald (Rice)
09:30 Coffee Break ( 00:30 )
10:00 Net Particle Higher Moments - Gary Westfall (Michigan State)
10:30 Mix Ratios - Lizhu Chen (IPP Wuhan)