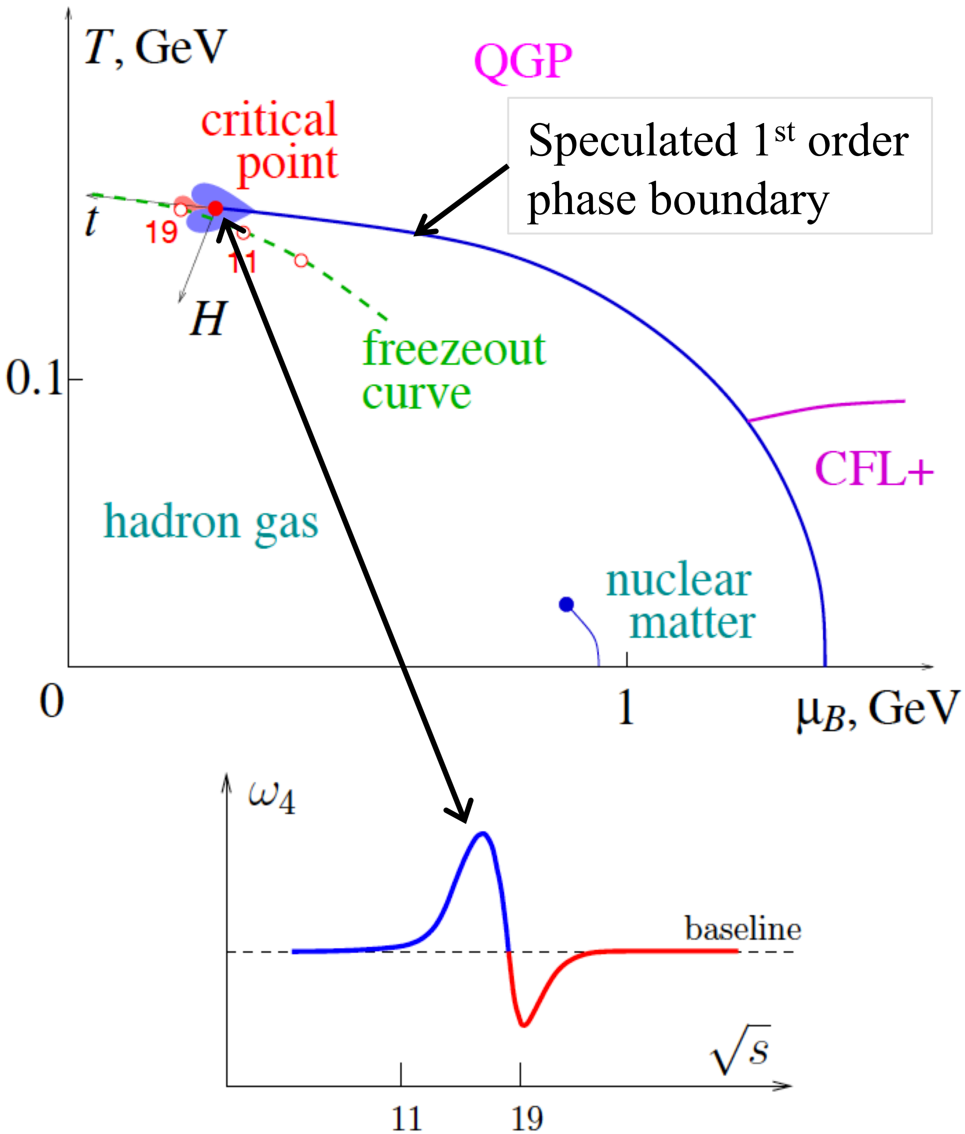
Extracting (μ_B,T) from Cumulants of Multiplicity Distributions

w.j. llope

Bulk Correlations PWG meeting, Nov 27, 2013

M. Stephanov, Rice Workshop, May 23-25, 2012



In the NLSM, experimentally-measured moments products are proportional to powers of the correlation length (critical opalescence)

Divergent values may indicate the Critical Point

It has thus been popular to measure the shapes of multiplicity distributions, as quantified by the moments, **μ, σ^2, S, K**, to search for the CP.

Decreased $1/VT^3$ dependence via $S\sigma = C_3/C_2$, $K\sigma^2 = C_4/C_2$

There is another analysis direction based on the multiplicity distribution shape information that can be pursued, and so far this direction is underexplored in STAR...

Use the cumulants to infer (μ_B,T)...

Are our net-p and net-q results "consistent"?

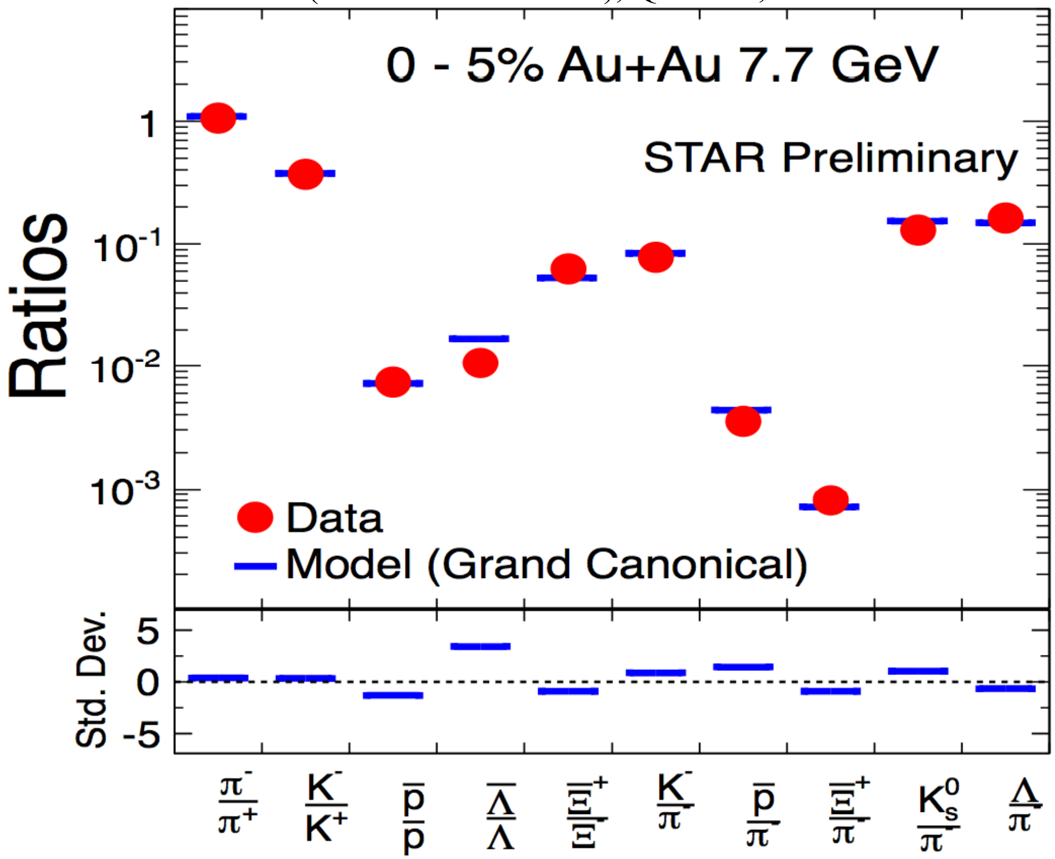
Have we sculpted the net-p and net-q results via the different cuts sets that we use for each?

Statistical Hadronization

The "standard" approach to infer (μ_B,T) from a data sample involves statistical hadronization models, such as THERMUS.

S. Wheaton *et al.*, Comp. Phys. Comm., **180**, 84 (2009)

S. Das (STAR Collaboration), QM2012, arXiv:1210.6099v1



At one √sNN & centrality:
Measure the ratios of efficiency-corrected average multiplicities (C_1) of identified particles in a specific kinematic region (|y|<0.1 for light hadrons)

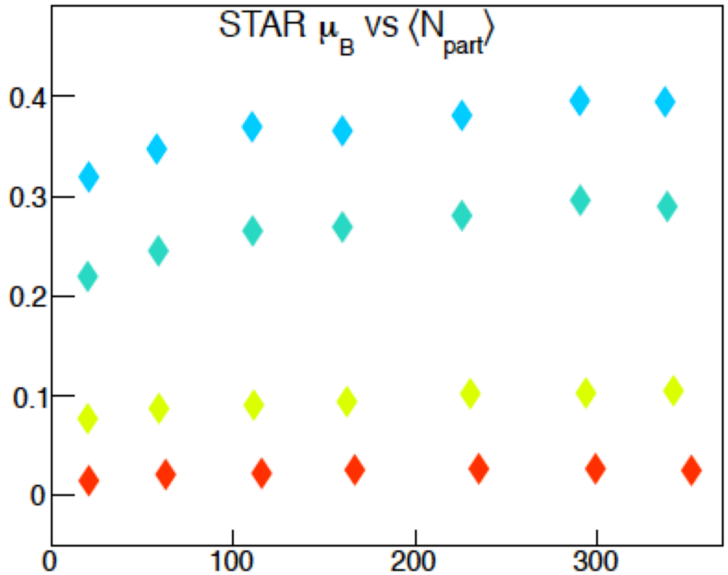
Then assume a (grand, strangeness, micro) canonical ensemble.

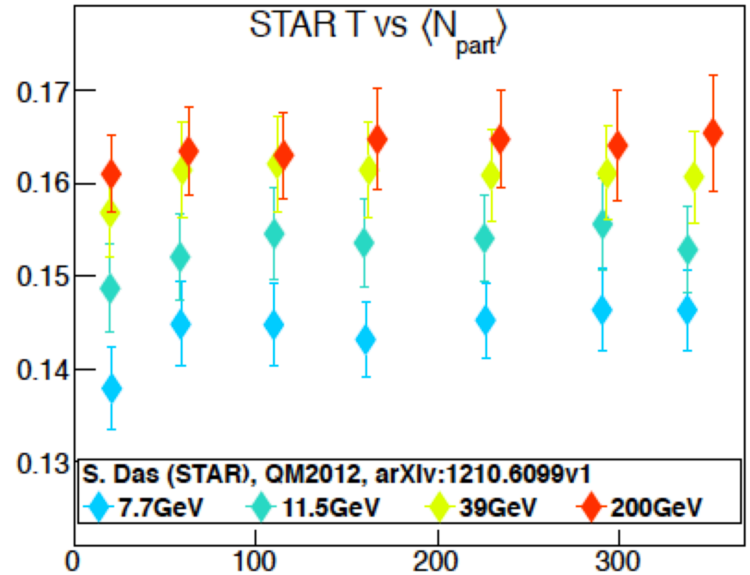
That assumption then allows the fitting of the measured ratios to functions that have some free parameters:

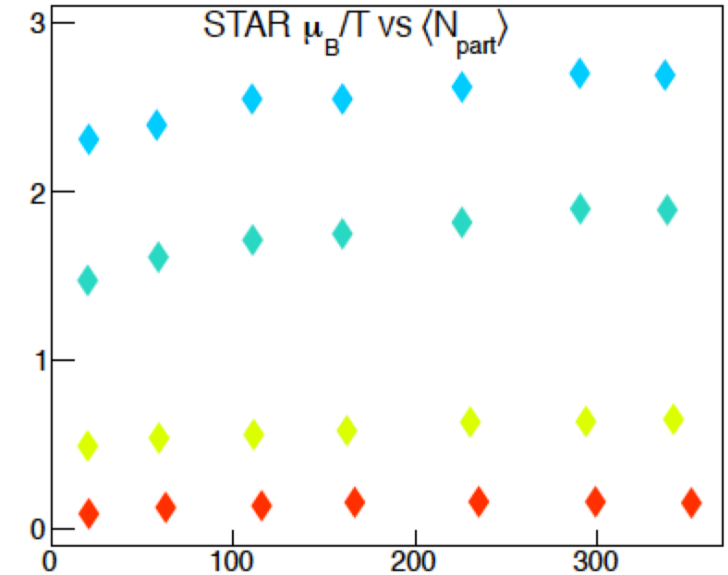
μ_B, T, γ_S, γ_Q, R, *etc*...

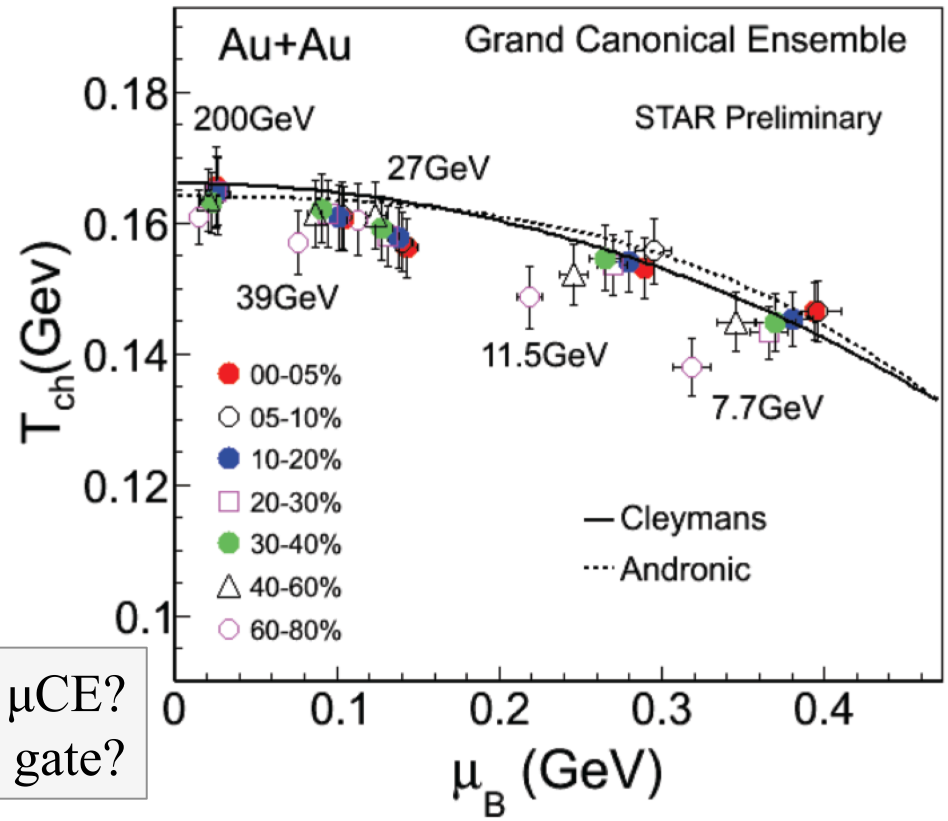
Applicability of the approach evaluated by χ^2, use of other ensembles, *etc.*

STAR SHM results









Centrality dependence in GCE *vs.* SCE *vs.* μCE?
Effects of wider rapidity gate?

Ratios of Multiplicity Distribution Cumulants: $R_{xy} = C_x/C_y$

We have very mature results on the net-p and net-q multiplicity distribution cumulants.
Use these plus Lattice QCD to infer (μ_B,T).

A. Bazavov, *et al.* (HotQCD), Phys. Rev. Lett., **109**, 192302 (2012)

Frithjof Karsch, University of Houston Colloquium, Sept. 24, 2013

Determination of T and μ_B from cumulant ratios

– in thermal equilibrium any two ratios of cumulants should allow to fix temperature and baryon chemical potential ♥

$$R^X_{n,m} = \frac{\chi^X_{n,\mu}}{\chi^X_{m,\mu}}\ , \quad X = B,\ Q,\ S$$

NLO Taylor expansion

– ratios with n+m even or odd show different sensitivity to T and μ_B

$$R^X_{12} \equiv \frac{M_X}{\sigma^2_X} = \frac{\mu_B}{T}\left(R^{X,1}_{12} + R^{X,3}_{12}\left(\frac{\mu_B}{T}\right)^2 + \mathcal{O}(\mu_B^4)\right)\ ,$$

$$R^X_{31} \equiv \frac{S_X\sigma^3_X}{M_X} = R^{X,0}_{31} + R^{X,2}_{31}\left(\frac{\mu_B}{T}\right)^2 + \mathcal{O}(\mu_B^4)\ ,$$

$M_X \sim \chi^X_1$: mean

$\sigma^2_X \sim \chi^X_2$: variance

$S_X \sim \chi^X_3/(\chi^X_2)^{3/2}$: skewness

♥ if fluctuations are sensitive to equilibrium physics at a unique (T, μ_B) pair

Ratios of Multiplicity Distribution Cumulants: $R_{xy} = C_x/C_y$

S. Mukherjee, WWND 2013

baryometer, fixes μ_B^f

LO linear in μ_B

$C_1/C_2 \rightarrow$

$$\frac{M_Q(\sqrt{s})}{\sigma_Q^2(\sqrt{s})} = \frac{\langle N_Q \rangle}{\langle (\delta N_Q)^2 \rangle} = \frac{\chi_1^Q(T,\mu_B)}{\chi_2^Q(T,\mu_B)} = R_{12}^{Q,1}(T)\mu_B + R_{12}^{Q,3}(T)\mu_B^3 + \cdots = R_{12}^Q(T,\mu_B)$$

$C_3/C_1 \rightarrow$

$$\frac{S_Q(\sqrt{s})\sigma_Q^3(\sqrt{s})}{M_Q(\sqrt{s})} = \frac{\langle (\delta N_Q)^3 \rangle}{\langle N_Q \rangle} = \frac{\chi_3^Q(T,\mu_B)}{\chi_1^Q(T,\mu_B)} = R_{31}^{Q,0}(T) + R_{31}^{Q,2}(T)\mu_B^2 + \cdots = R_{31}^Q(T,\mu_B)$$

thermometer, fixes T^f

LO independent of μ_B

STAR, PHENIX

HIC

mean: M_Q

variance: σ_Q^2

skewness: S_Q

$\delta N_Q = N_Q - \langle N_Q \rangle$

LQCD

generalized charge susceptibilities:

$$\chi_n^Q(T,\vec{\mu}) = \frac{1}{VT^3}\frac{\partial^n \ln Z(T,\vec{\mu})}{\partial(\mu_Q/T)^n}$$

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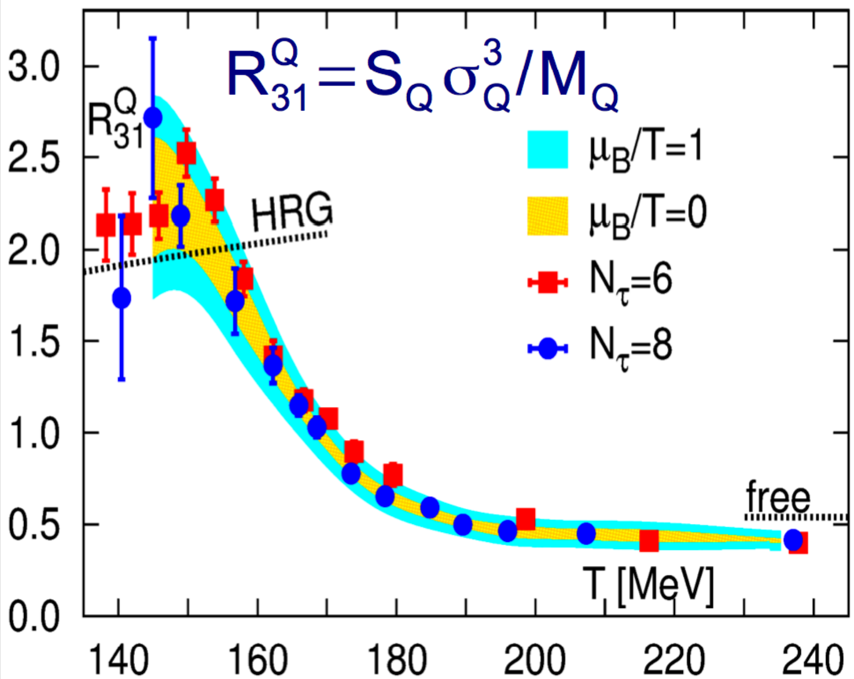
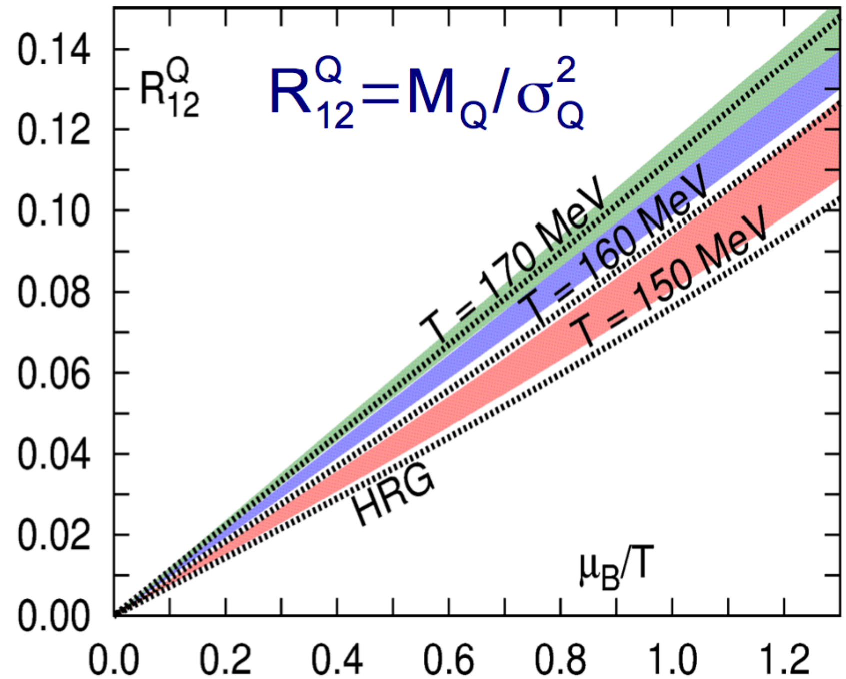
Basic approach: Measure R_{12} and R_{31}, then pick off μ_B/T and T from the Lattice results.

S. Mukherjee, WWND 2013





$S_Q \sigma^3_Q / M_Q$	T^f [MeV]
$\gtrsim 2$	$\lesssim 155$
~ 1.5	~ 160
$\lesssim 1$	$\gtrsim 170$

M_Q / σ^2_Q	μ^f_B / T^f
0.01 – 0.02	0.1 – 0.2
0.03 – 0.04	0.3 – 0.4
0.05 – 0.08	0.5 – 0.7

BNL-BI: PRL 109, 192302 (2012)

for: $T^f \sim 160$ MeV

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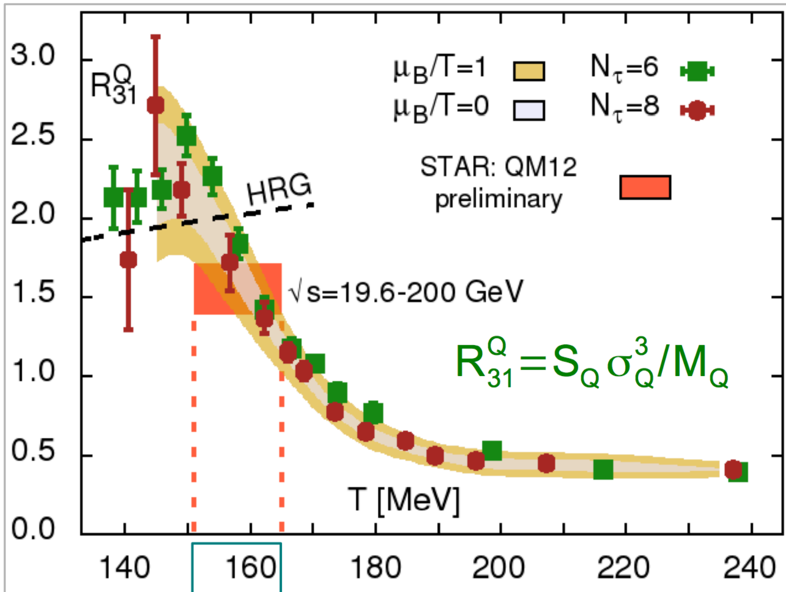
Results when using QM2012 net-q cumulants



variation of $T^{f,ch}$ is < 5 MeV for $\sqrt{s_{NN}} > 19$ GeV

as a first start, use the average value over

$\sqrt{s_{NN}} = 19.6 - 200$ GeV

$\overline{R_{31}^Q} = 1.56(16)$

STAR preliminary
Quak Matter 2012

$T^f = 158(7)$ MeV

for orientation:

$T_c(\mu_B = 0) = 154(9)$ MeV

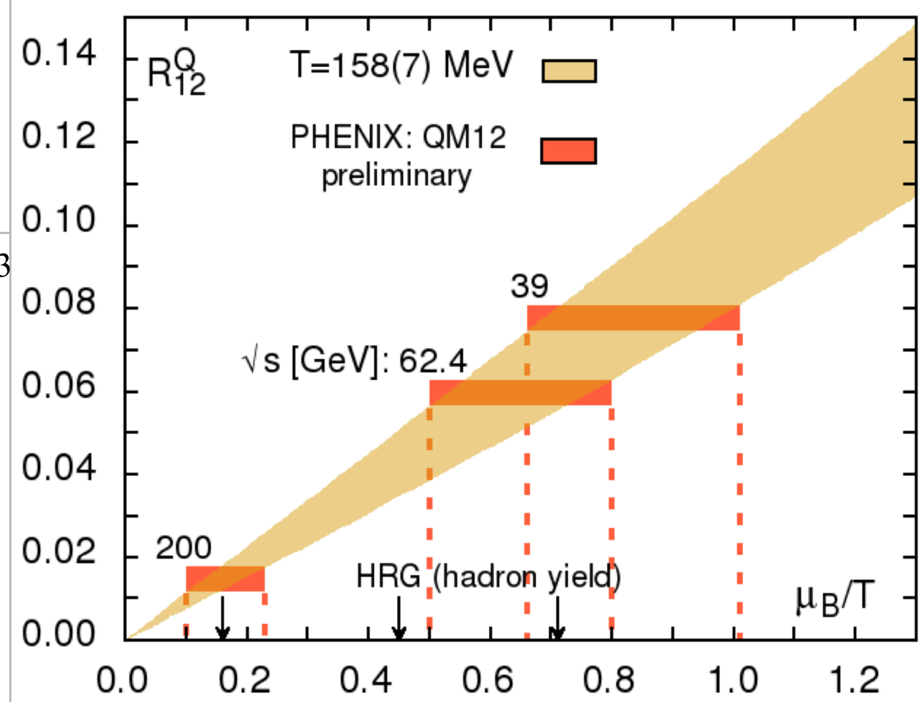
$T^{f,ch} = 160(5)$ MeV

S. Mukherjee, WWND 2013

Thermometer from R_{13}^Q

Baryometer from R_{21}^Q

$R_{12}^Q = M_Q / \sigma_Q^2$



μ_B^f / T^f agree reasonably with $\mu_B^{f,ch} / T^{f,ch}$

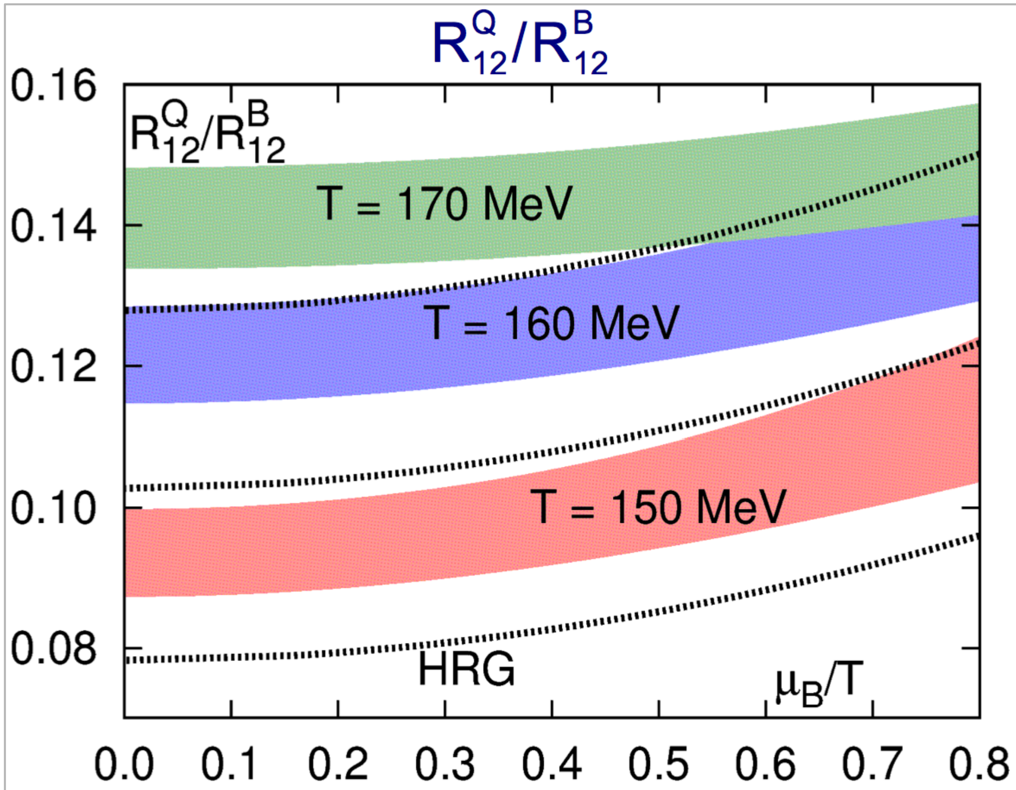
Thermometer:
Reasonable agreement to lattice Tc = 154(9) MeV

Baryometer:
Fair agreement of PHENIX data to HRG

Thermodynamic consistency

The values of (μ_B,T) from R_{xy}^B should be consistent with those from R_{xy}^Q

S. Mukherjee, WWND 2013



if the fluctuations are described by equilibrium thermodynamics

R^Q_{12} and R^B_{12} must contain identical information regarding T and μ_B

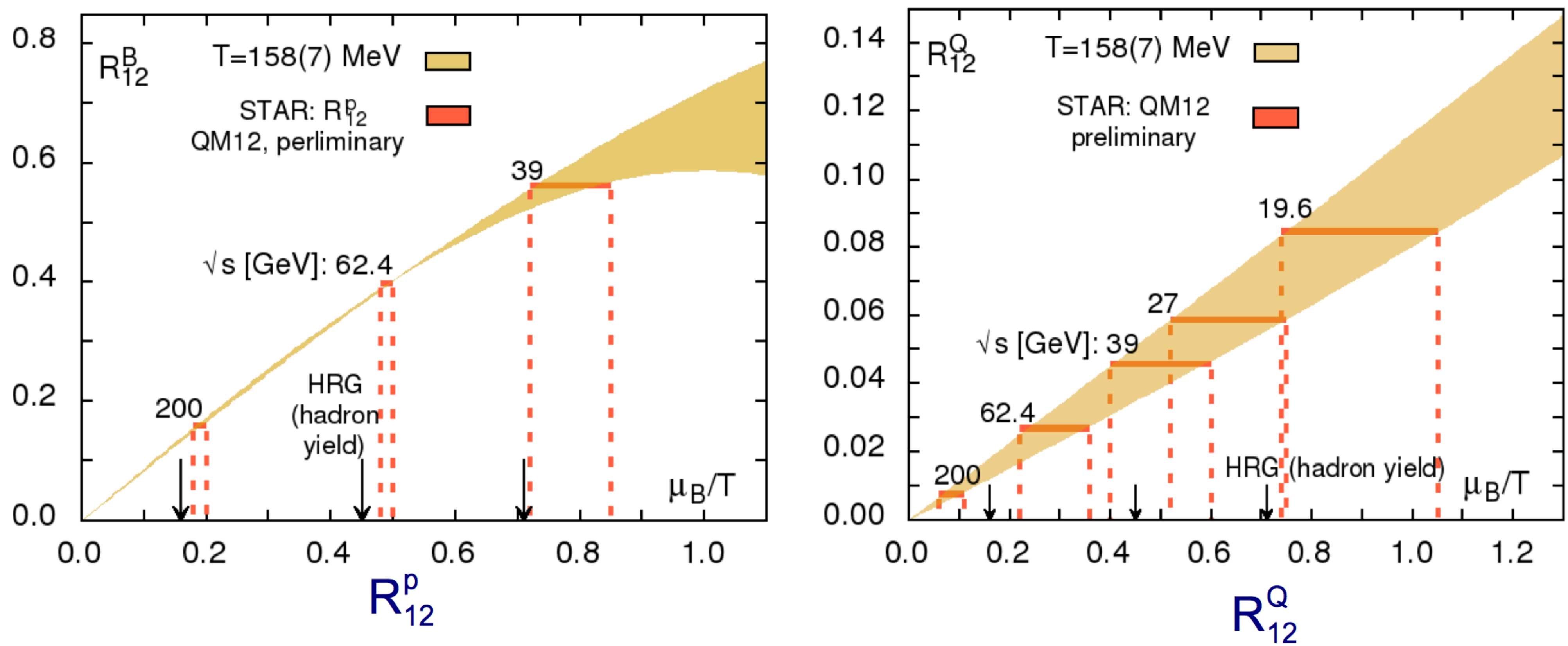
However ...

currently STAR preliminary @ $\sqrt{s_{NN}}=200$ GeV: $R^Q_{12}/R^p_{12} \approx 0.06$

a problem !!

Consistency

S. Mukherjee, WWND 2013



give inconsistent values for μ_B^f

a problem !!

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The Wuppertal-Budapest LQCD group has also recently investigated this direction

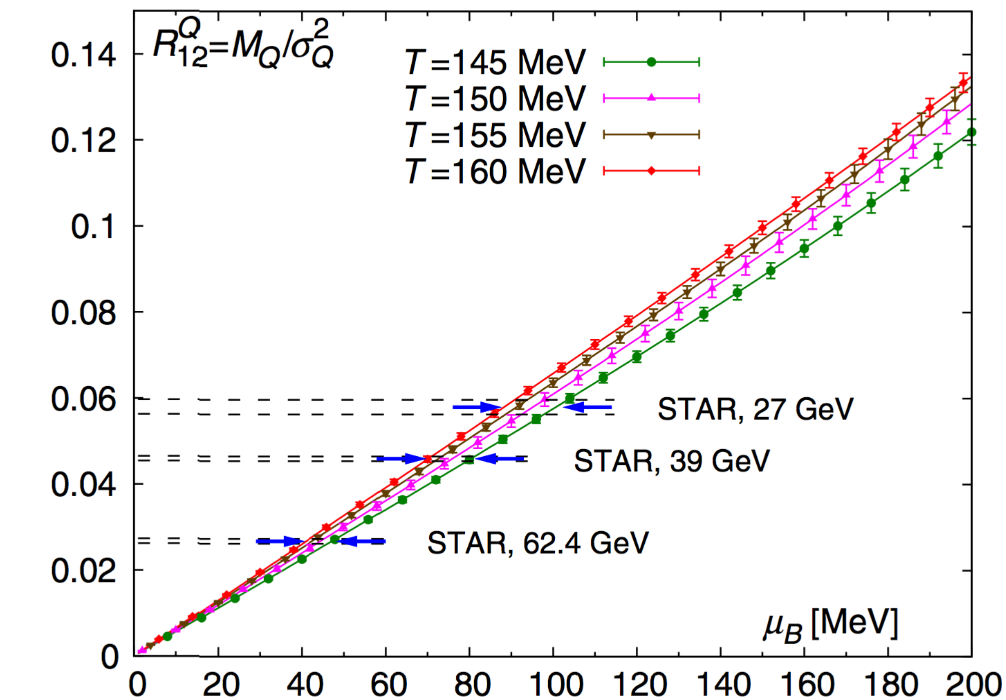


FIG. 4 (color online). R^Q_{12} as a function of μ_B: the different colors correspond to the continuum extrapolated lattice QCD results, calculated at different temperatures. The three points correspond to preliminary STAR data for M_Q/σ^2_Q at different collision energies: $\sqrt{s}$ = 27, 39, 62.4, from Ref. [6].

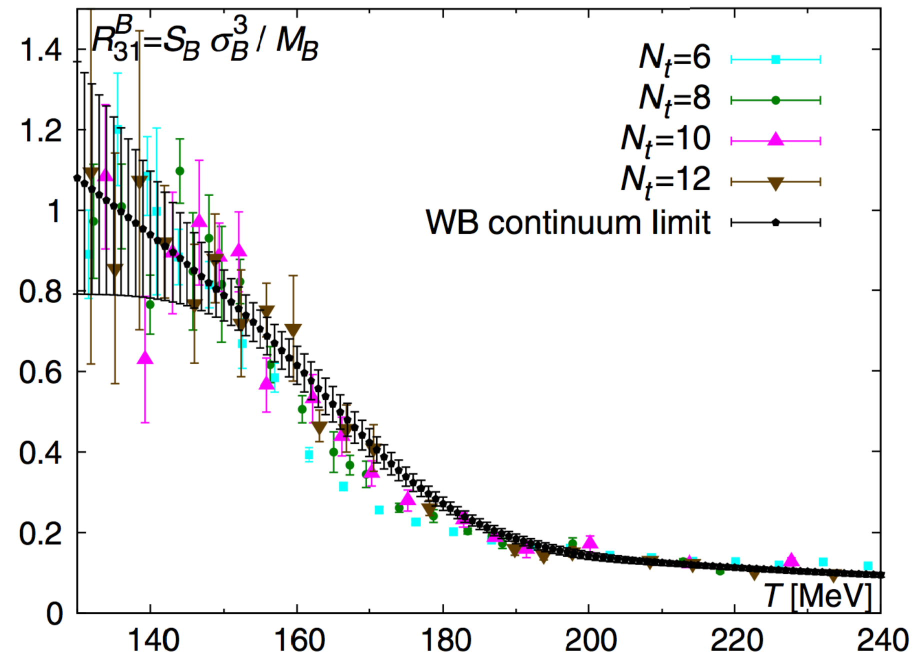


FIG. 5 (color online). R^B_{31}: the colored symbols correspond to lattice QCD simulations at finite N_t. The black points correspond to the continuum extrapolation.

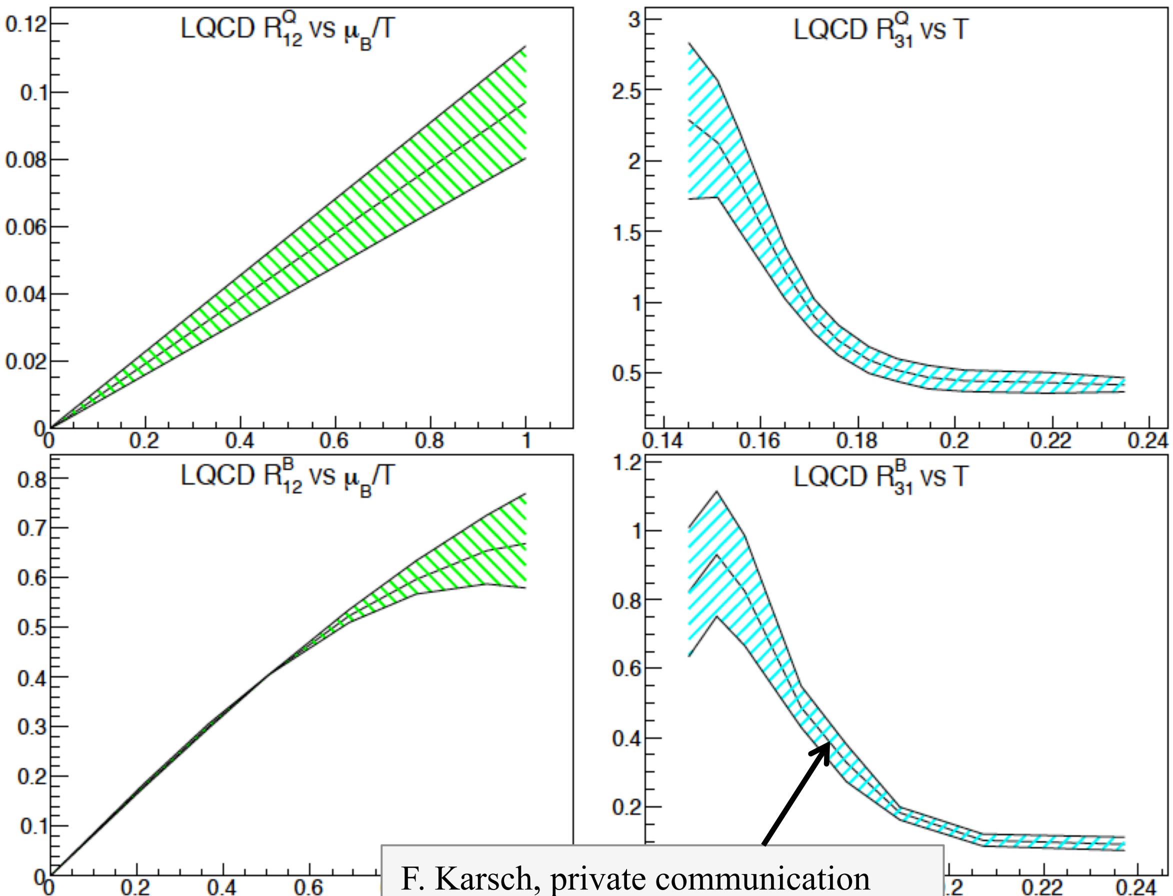
TABLE I. Freeze-out baryon chemical potentials vs the corresponding collision energy of the three STAR measurements from Ref. [6]. The errors come from the uncertainty of the freeze-out temperature, the lattice statistics, and the experimental error,

$\sqrt{s}$ [GeV]	μ_B^f [MeV]
62.4	44(3)(1)(2)
39	75(5)(1)(2)
27	95(6)(1)(5)
	$()_{\delta T}()_{\text{lat}}()_{\text{exp}}$

S. Borsányi *et al.* (Wuppertal-Budapest), Phys. Rev. Lett. **111**, 062005 (2013)

My goals

Now we have efficiency-corrected values!
Effect of kinematic acceptance (different for net-p and net-q)?



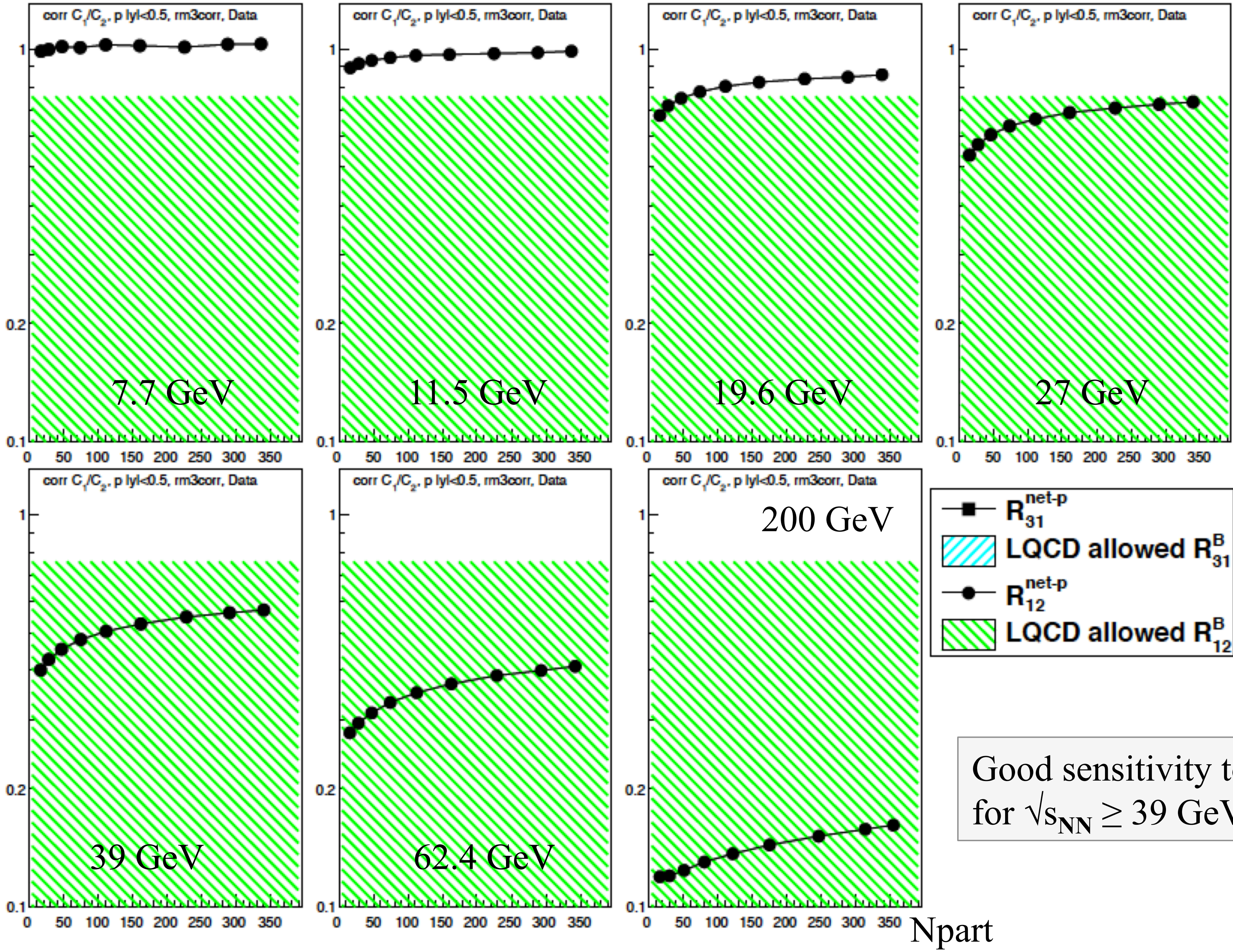
F. Karsch, private communication
Do Not Circulate

net-p
refmult3corr (π&K, |y|<1.0)
|y|<0.5, 0.4<P_T<0.8
n_σ(p)<2
Nhitsfit>20, no Nhitsdedx cut
DCAglobal<1
|ZvtxTPC-ZvtxVPD|<3 (≥39 GeV)

net-q
refmult2corr (chgd, 0.5<|η|<1)
|η|<0.5, 0.2<P_T<2.0 -spallation p
Nhitsfit>20, Nhitsdedx>10
DCAglobal<1
|ZvtxTPC-ZvtxVPD|<4 (≥39 GeV)

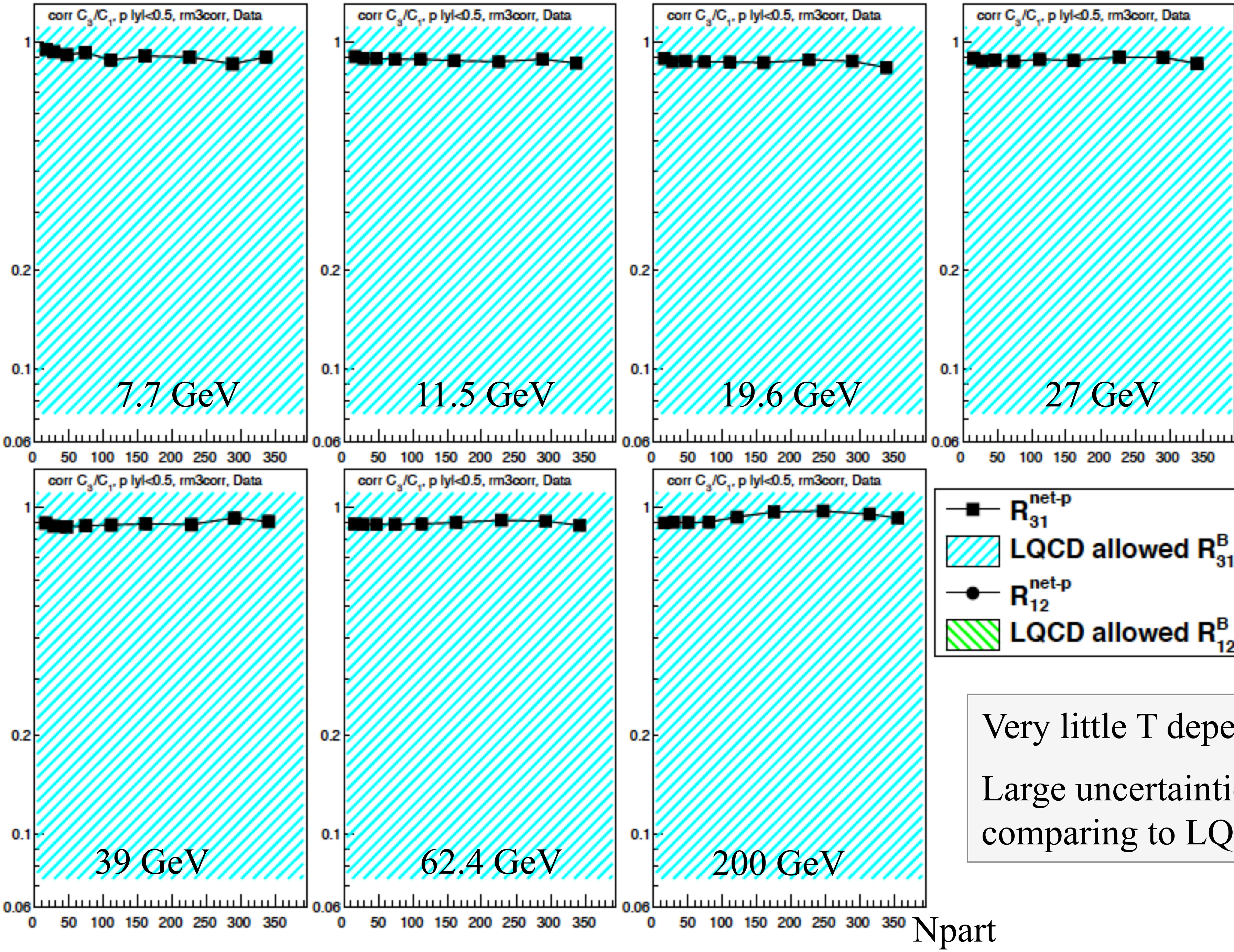
1. Use our latest **efficiency-corrected** results for $R_{xy}^{B,Q}$ to extract (μ_B,T)
2. Produce new values of $R_{xy}^{B,Q}$ using different centrality definitions to allow **more consistent kinematic acceptances** for R_{xy}^{B} and R_{xy}^{Q}

R_{12}=C_1/C_2, **net-p**, |y|<0.5, refmult3corr(|y|<1)



Good sensitivity to μ_B/T for $\sqrt{s_{NN}} \geq 39$ GeV

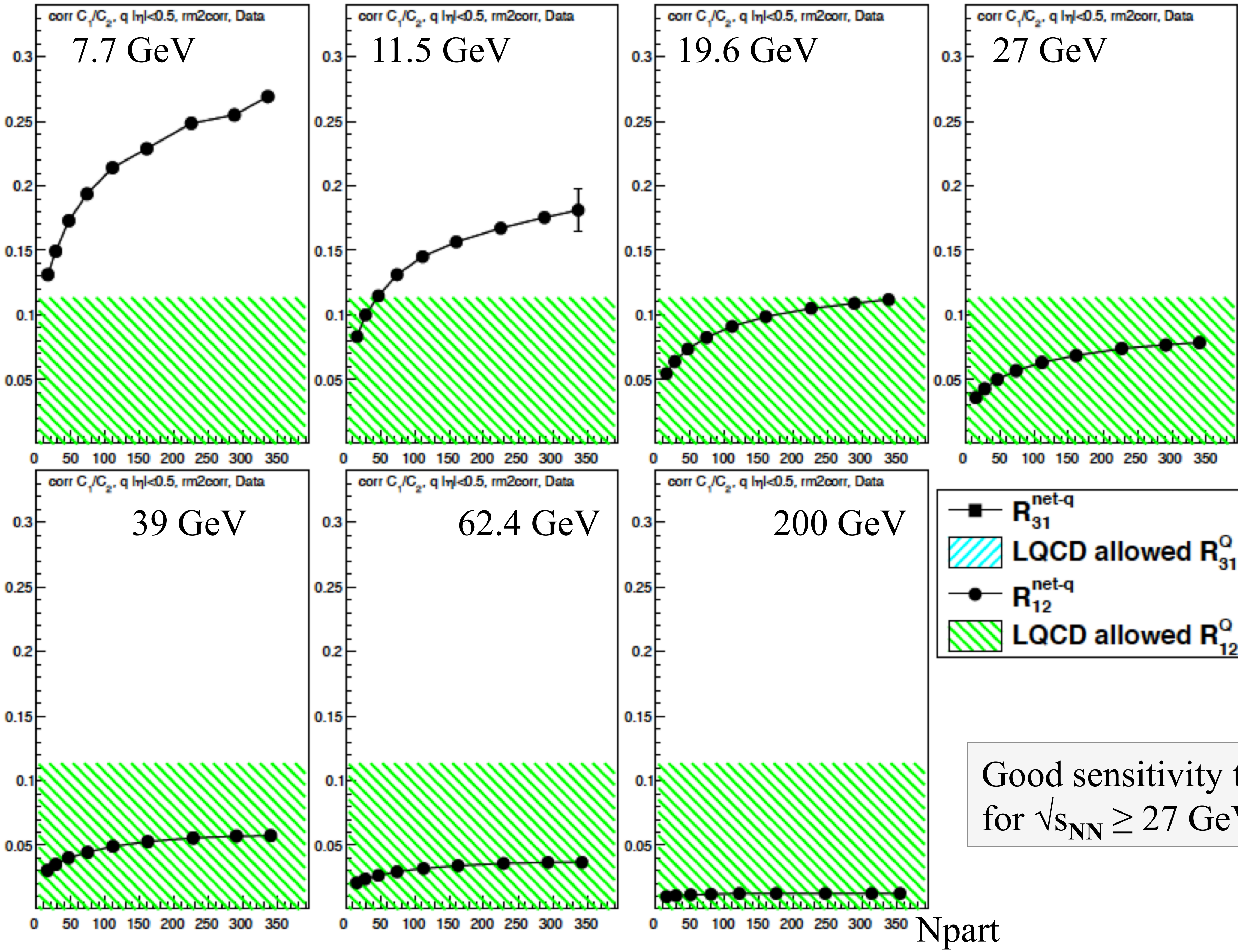
$R_{31}=C_3/C_1$, **net-p**, |y|<0.5, refmult3corr(|y|<1)



Very little T dependence.

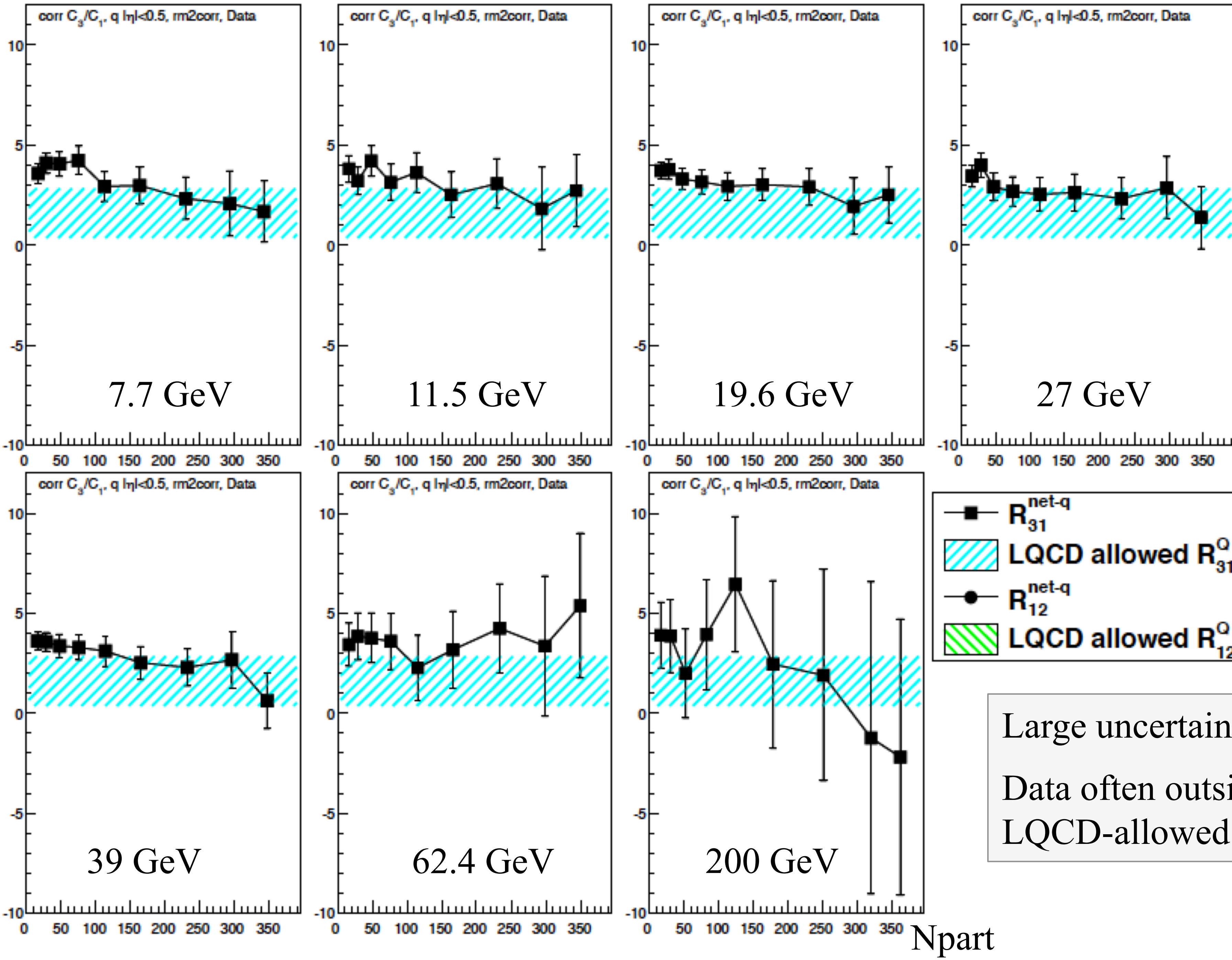
Large uncertainties when comparing to LQCD.

$R_{12}=C_1/C_2$, **net-q**, $|\eta|<0.5$, refmult2corr($0.5<|\eta|<1$)



Good sensitivity to μ_B/T for $\sqrt{s_{NN}} \geq 27$ GeV

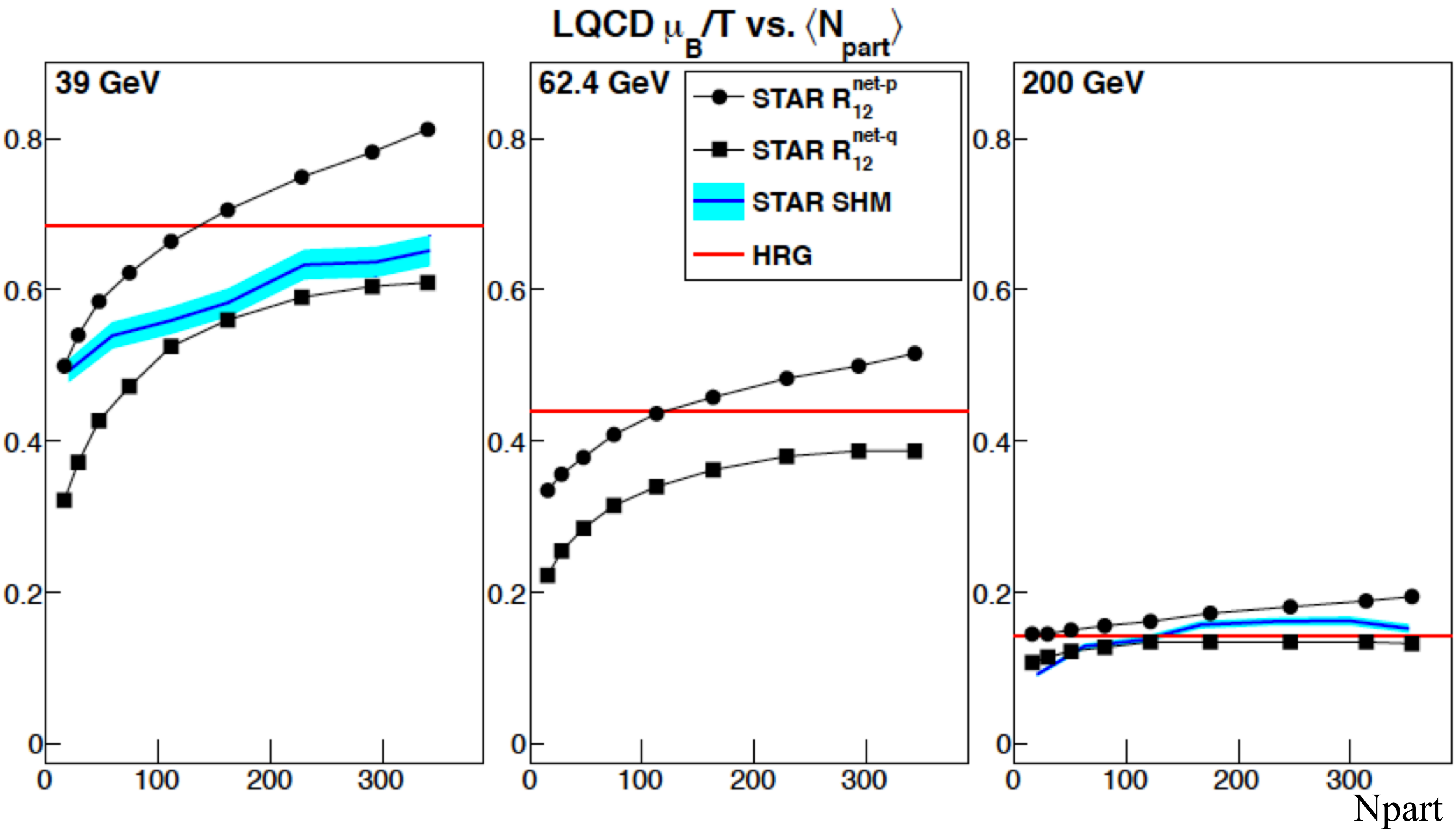
$R_{31}=C_3/C_1$, **net-q**, $|\eta|<0.5$, refmult2corr($0.5<|\eta|<1$)



Large uncertainties.

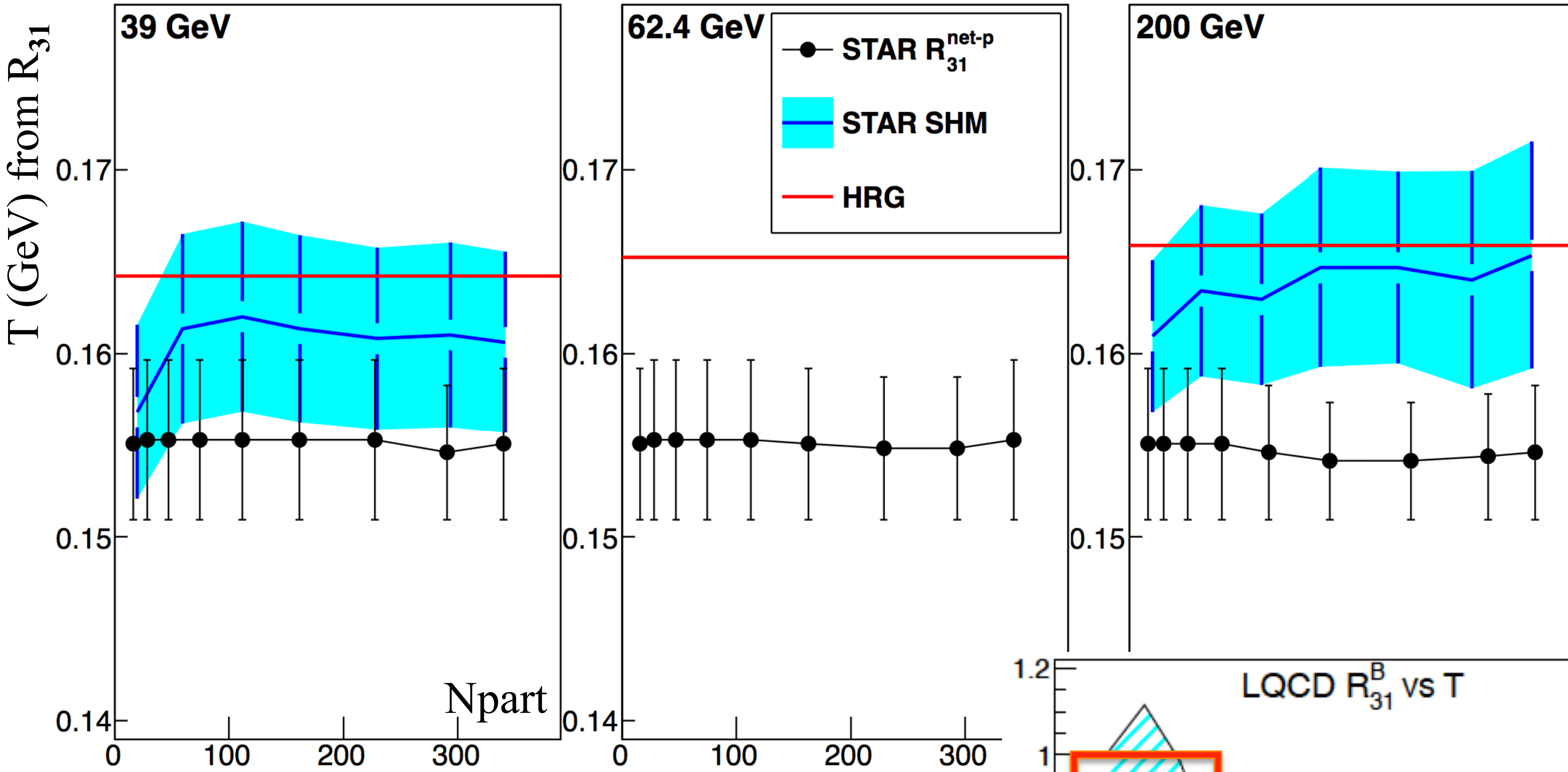
Data often outside LQCD-allowed area.

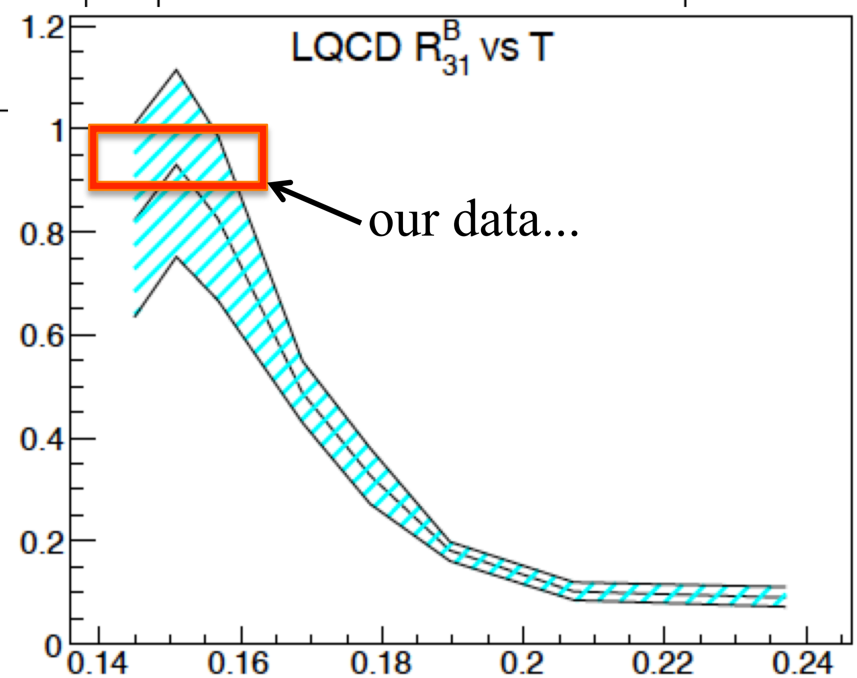
net-p Cumulant+LQCD *vs.* net-q Cumulant+LQCD *vs.* SHM



Cumulants+LQCD imply μ_B/T decreases as centrality decreases (similar to SHM w/ GCE)
μ_B/T from net-p and net-q diverge as $\sqrt{s_{NN}}$ decreases.
μ_B/T from net-p > μ_B/T from net-q
SHM results similar to the Cumulants+LQCD values (in between net-p & net-q)

Temperature from R_{31}, Net-p+LQCD





Net-q values are all over the map w.r.t. LQCD allowed range...

Net-p values allow an extraction of T from R31Q, but not with much sensitivity...

Intermediate summary

Summary so far:

Used latest efficiency-corrected net-p and net-q moments products to constrain μ_B/T & T using LQCD predictions. This is an alternative to SHM approaches...

- Reasonable sensitivity to μ_B/T from $R_{12}^{net\text{-}p}$ and $R_{12}^{net\text{-}q}$...
- Not much sensitivity to T from $R_{31}^{net\text{-}p}$...
- Data for $R_{31}^{net\text{-}q}$ has large errors and are often outside the LQCD allowed range...

μ_B/T from $R_{12}^{net\text{-}p}$ & $R_{12}^{net\text{-}q}$ increases as the centrality increases...

- Similar to the centrality dependence from the STAR SHM results with the GCE...

μ_B/T from $R_{12}^{net\text{-}p}$ & $R_{12}^{net\text{-}q}$ are inconsistent, and become more so as $\sqrt{s_{NN}}$ decreases...

Is this a result of the different kinematic cuts used in the net-p and net-q analyses?

net-p	**net-q**
refmult3corr (π&K, |y|<1.0)	refmult2corr (chgd, 0.5<|η|<1)
|y|<0.5, 0.4<P_T<0.8	|η|<0.5, 0.2<P_T<2.0 -spallation p
n_σ(p)<2	
Nhitsfit>20, no Nhitsdedx cut	Nhitsfit>20, Nhitsdedx>10
DCAglobal<1	DCAglobal<1
|ZvtxTPC-ZvtxVPD|<3.0 (≥39 GeV)	|ZvtxTPC-ZvtxVPD|<4.0 (≥39 GeV)

To explore this, I need centrality definitions that do not use the TPC...

...and/or should not strongly autocorrelate with the total multiplicity of TPC primaries...

New centrality introduction

With a centrality definition that does not use the TPC, I can measure the net-p and net-q cumulants using the same centrality definition (✔), more similar kinematic cuts (✔), and a less restrictive psuedorapidity range for net-q (✔)...

Recall my earlier studies on alternative centrality definitions (2011):

http://wjllope.rice.edu/fluct/protected/cent_slides_20110817.pdf

http://wjllope.rice.edu/fluct/protected/cent_slides_20110914.pdf

At that time, I studied BEMC ΣE, and ZDC *vs.* BBC.

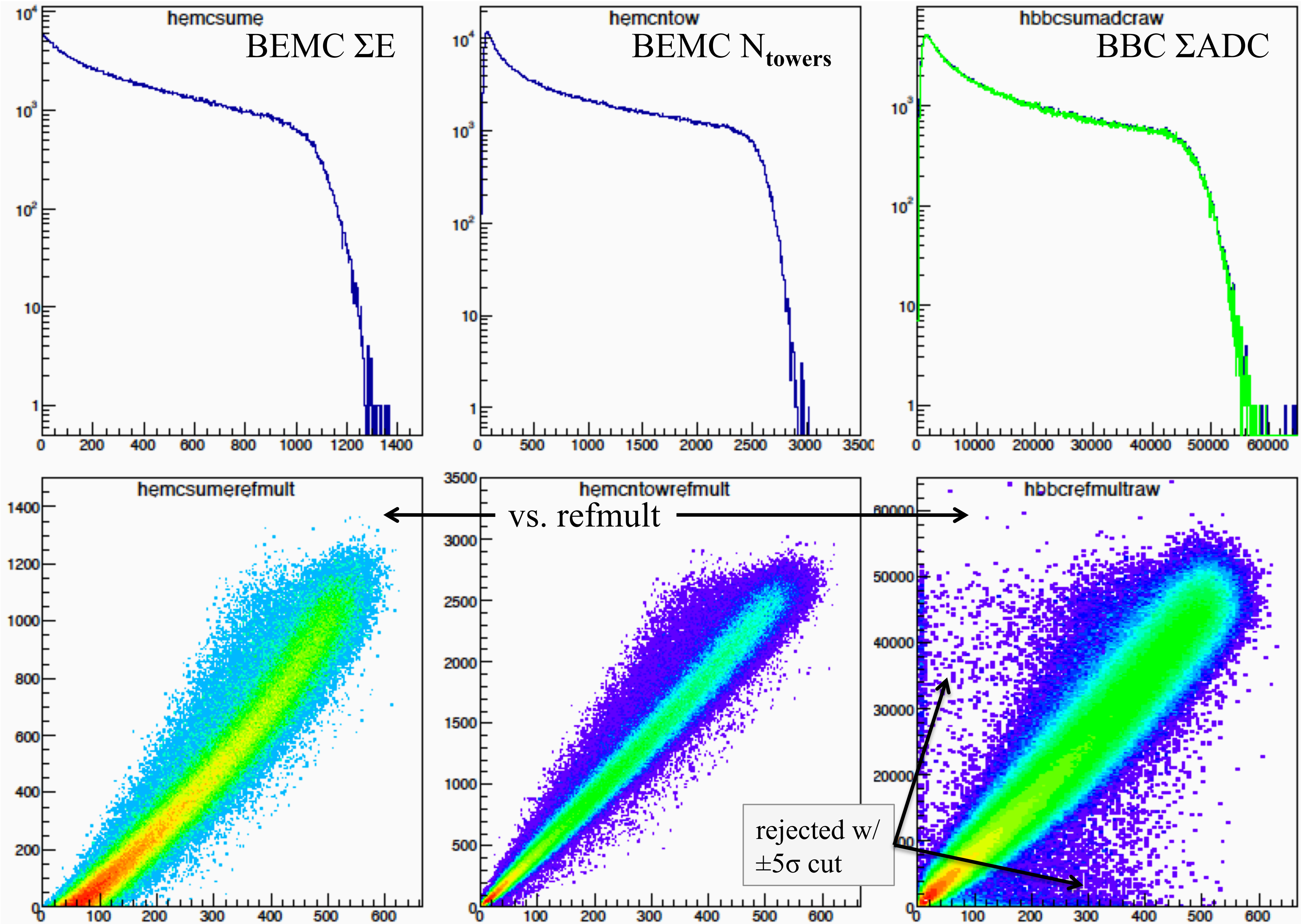
BEMC ΣE showed some energy scale jumps in the low-$\sqrt{s_{NN}}$ BES data, but at the moment I am most interested in the 200 GeV data, where the peds and gains are in good shape.

Will explore:

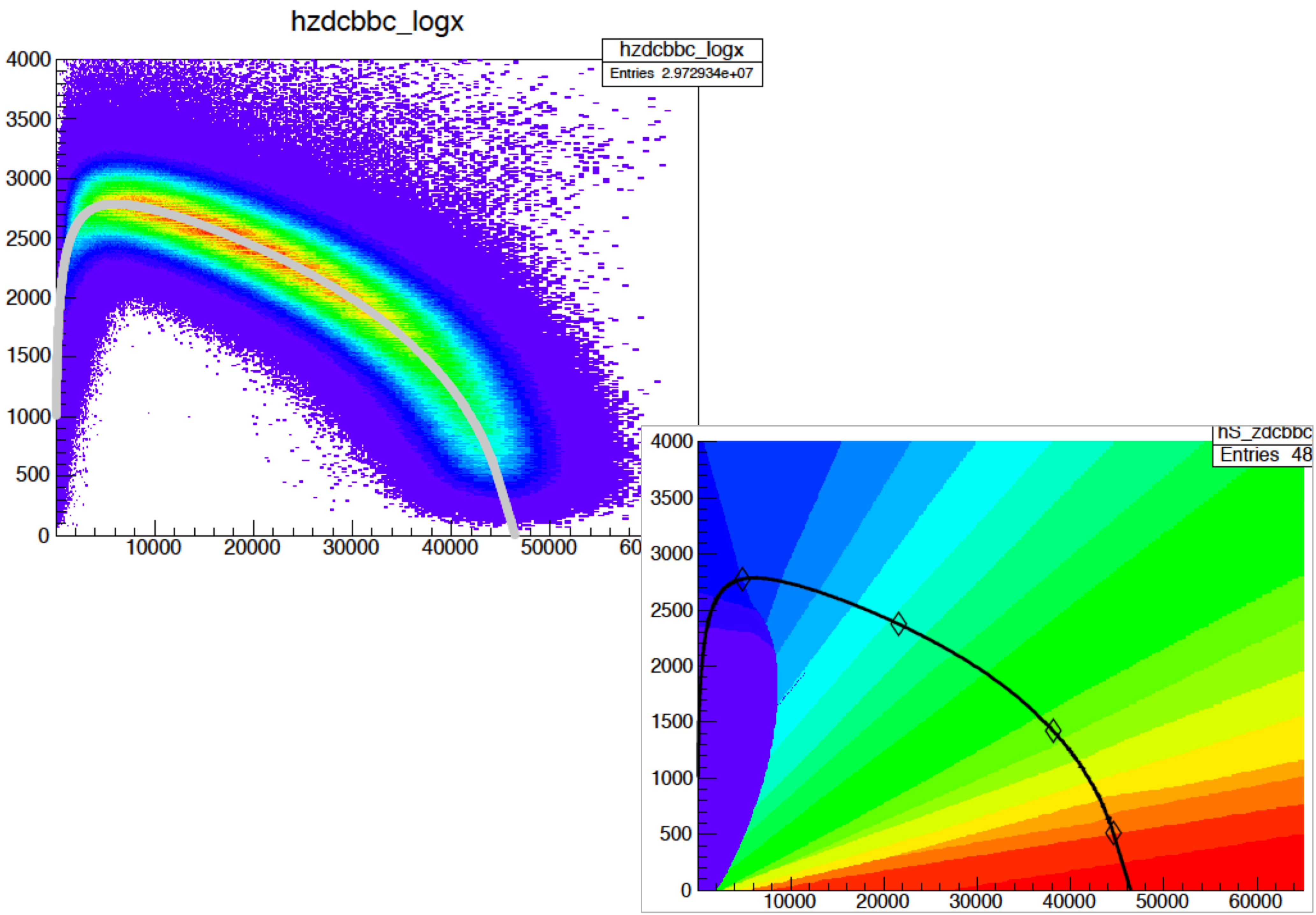
- BBC ΣADC
- BEMC ΣE
- BEMC N_{towers} (ADC > pedavg+4*pedrms)
- ZDC ΣADC *vs.* BBC ΣADC

Of course, this same code can also still use the "standard" refmultXcorr and the same cuts used in the net-p and net-q papers to check the consistency, and I can also explore the sensitivity of the results to different cuts and centrality selections.

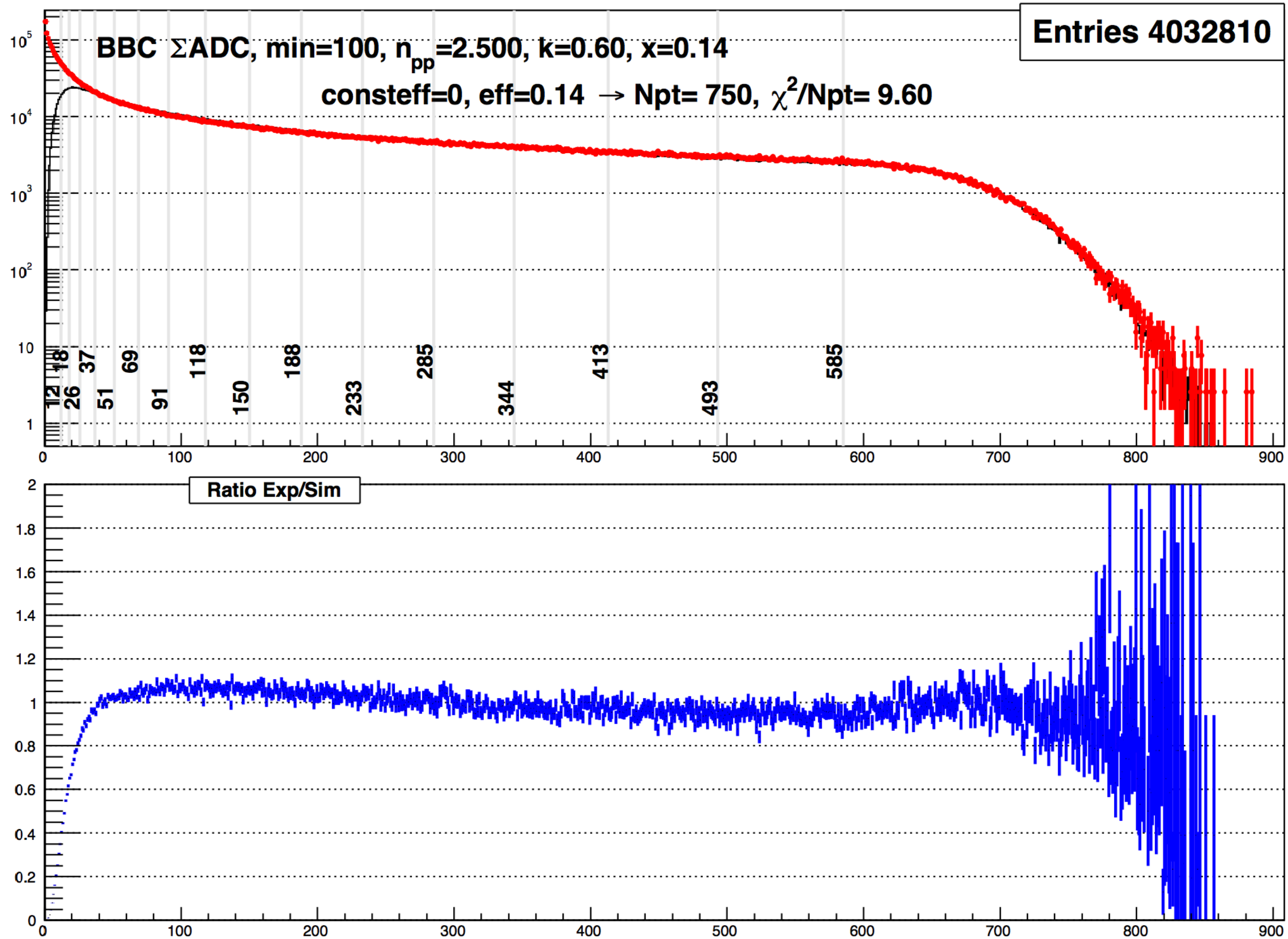
New !TPC centrality, 200 GeV



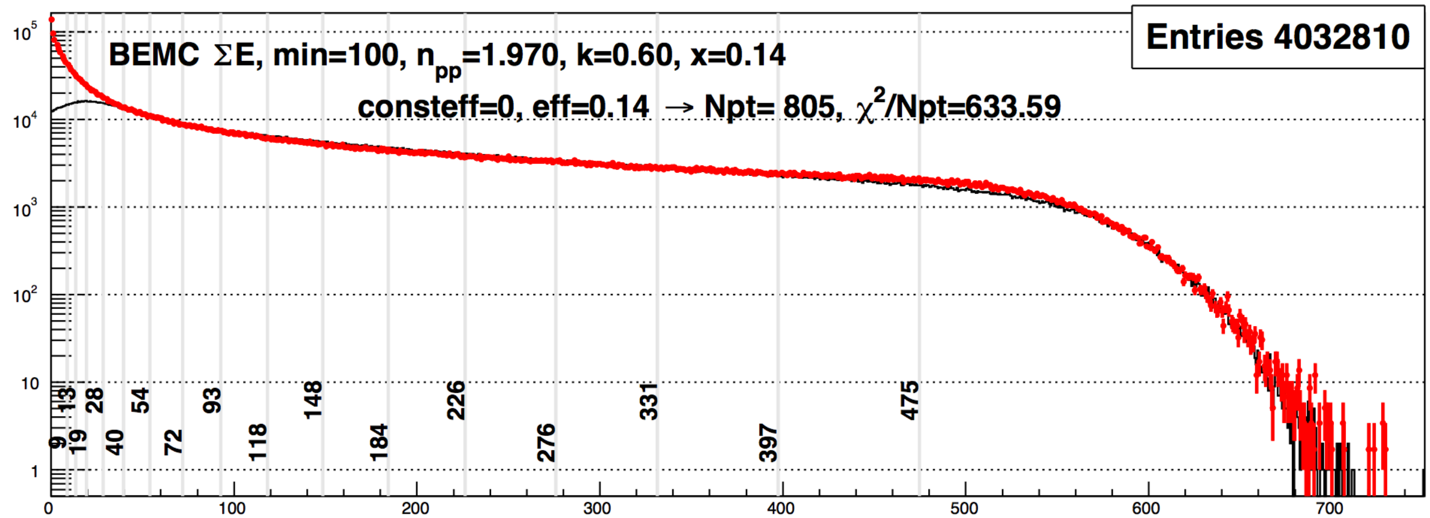
ZDC ΣADC *vs.* BBC ΣADC

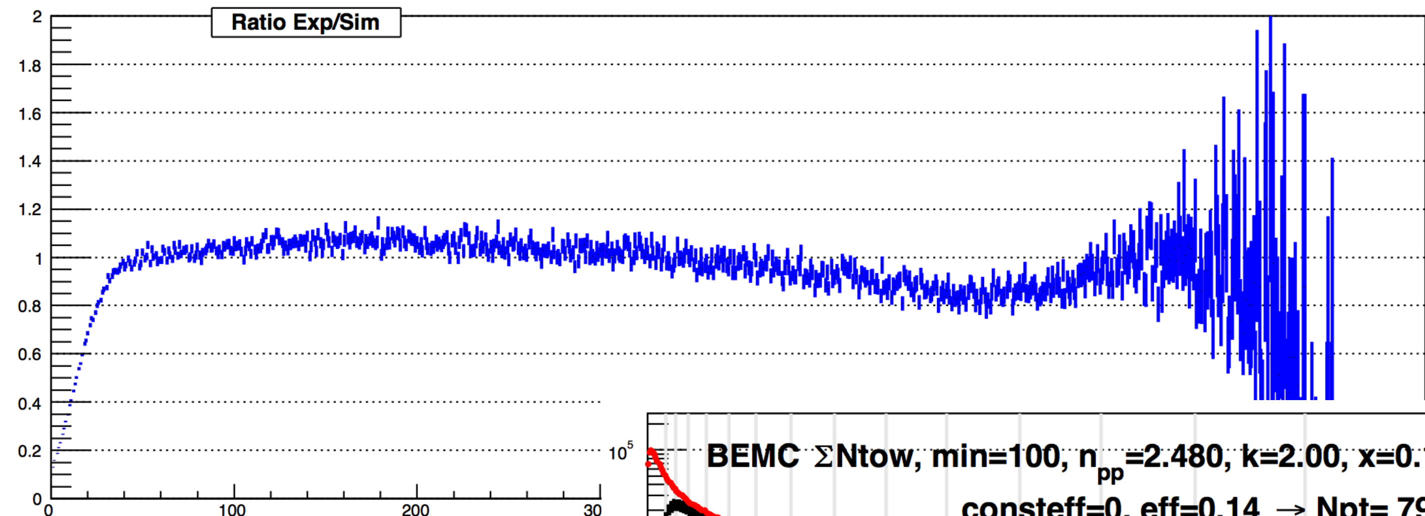


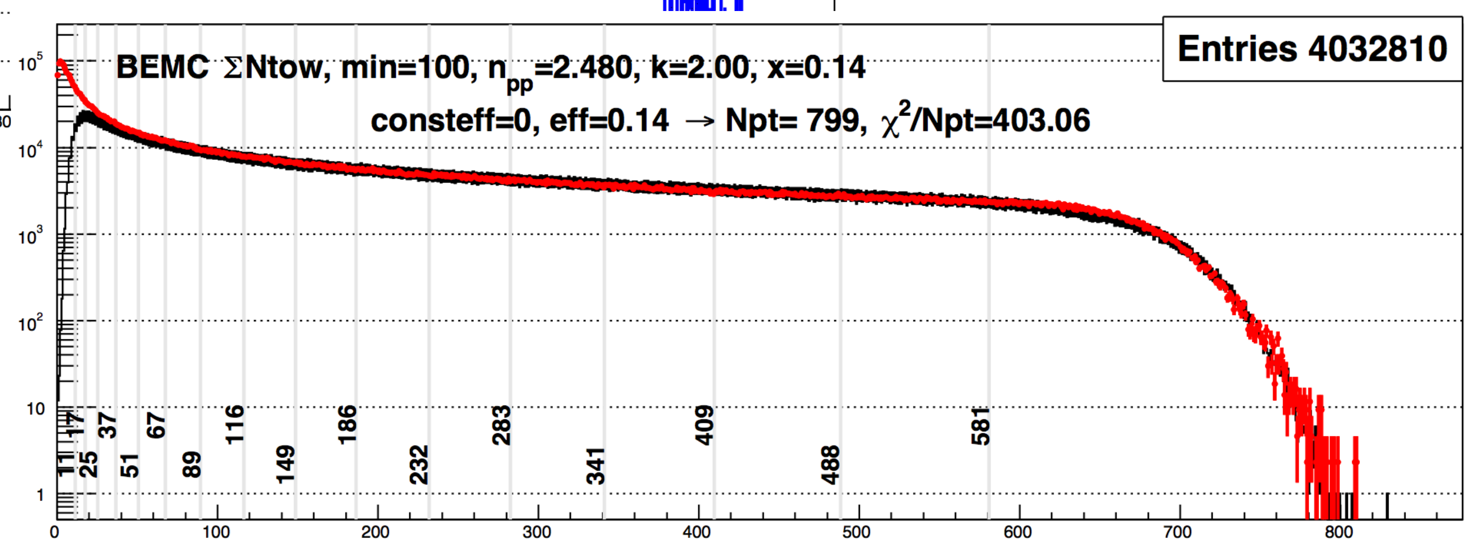
Glauber calculation for BBC ΣADC, 200 GeV



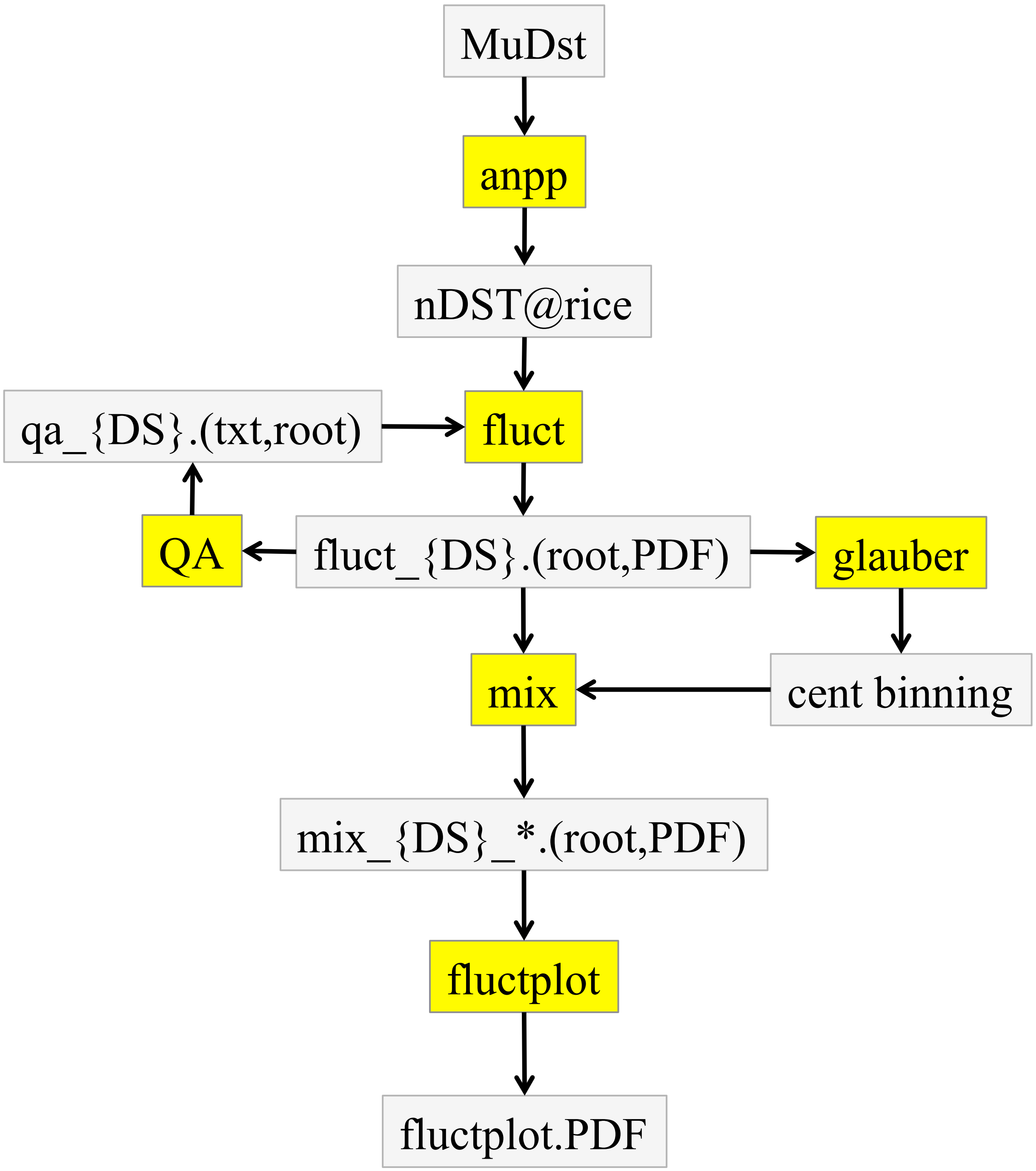
Glauber calculations for BEMC variables, 200 GeV







Codes



{DS} unique identifier for year and $\sqrt{s_{NN}}$
- Data
- Compiled C++ code

anpp:
select minbias trigger, apply |Zvtx| cut. calculate refmultX
save event info and all primary tracks to TTrees

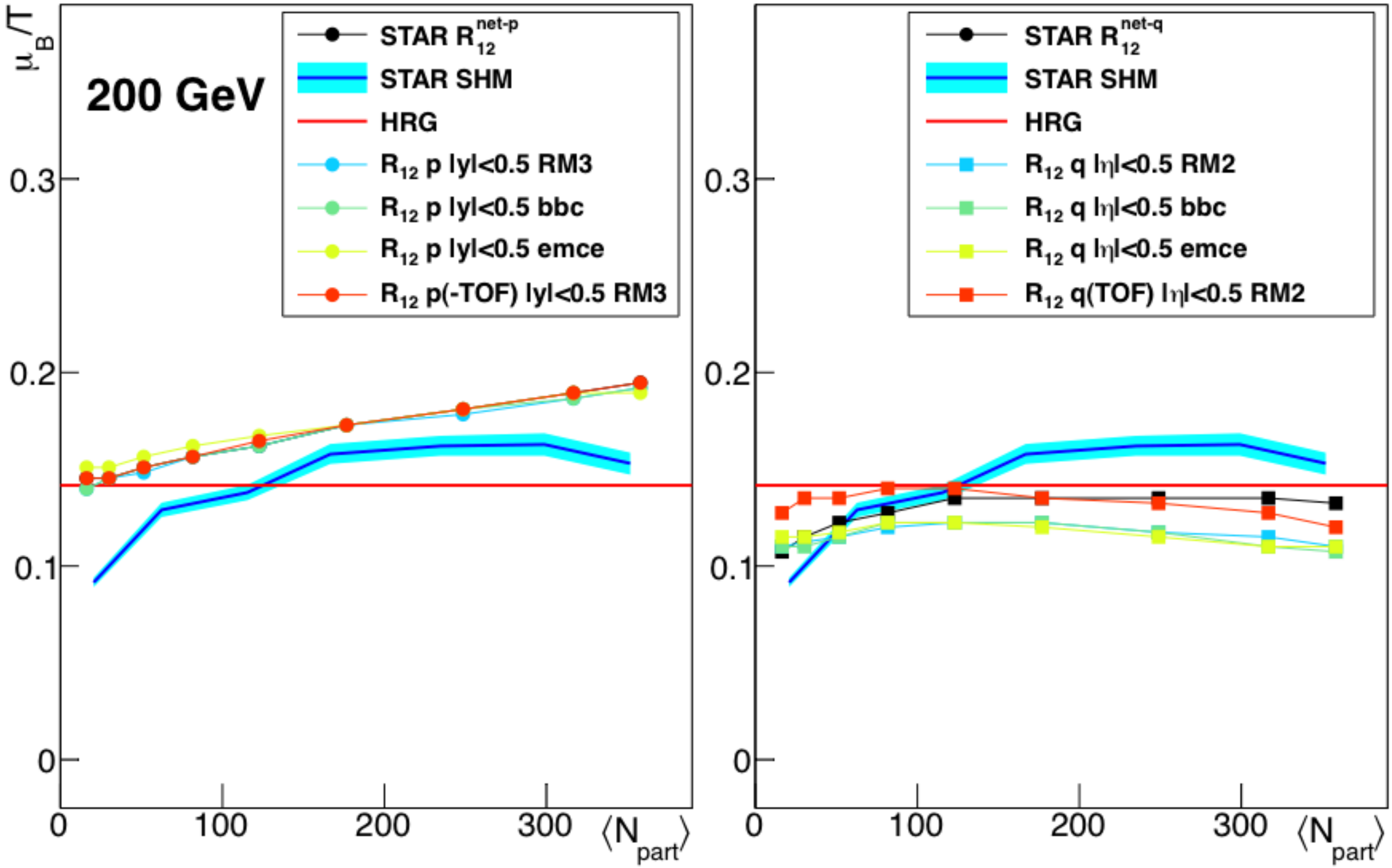
fluct:
fill 4 "base" TH2Ds for specific track cut sets (net,tot,pos,neg) vs. centrality variable

qa:
bad runs: 30 variables, check 6, require ≥4 vars fail
bad events: 10 2D correlation plots, check 2, ±Nσ cuts

mix:
read TH2Ds from net-p paper, net-q paper, or fluct
calculate Cx, Rxy *vs.* centrality variable
efficiency corrections
CBW averaging
bootstrap errors
Sampled singles/IRV cumulant arithmetic

fluctplot:
collect results from all sources and make final plots
make connections to LQCD

μ_B/T from fluct net-p and net-q



$R_{12}^{net\text{-}p}$ is quite stable *vs.* centrality variable used, $R_{12}^{net\text{-}q}$ is not…
fluct code reproduces net-p paper C_x and R_{12}, but not net-q paper C_x and R_{12}…

Summary

Used latest efficiency-corrected net-p and net-q moments products to constrain μ_B/T & T using LQCD predictions. This is an alternative to SHM approaches...

Reasonable sensitivity to μ_B/T from R_{12}^{net-p} and R_{12}^{net-q}

Not much sensitivity to T from R_{31}^{net-p}

Data for R_{31}^{net-q} has large errors and are often outside the LQCD allowed range

μ_B/T from R_{12}^{net-p} & R_{12}^{net-q} increases as the centrality increases...

Similar to the centrality dependence from the STAR SHM results with the GCE

μ_B/T from R_{12}^{net-p} & R_{12}^{net-q} are inconsistent, and become more so as $\sqrt{s_{NN}}$ decreases...

There are two recent PRLs from two major LQCD collaborations, who will soon use the new efficiency-corrected net-p and net-q paper results to constrain μ_B/T & T

Aside from the CP search, do the two moments papers tell a consistent story at high $\sqrt{s_{NN}}$?

Four new centrality definitions based on:

BBC ΣADC, BEMC ΣE, BEMC N_{towers}, ZDC ΣADC *vs.* BBC ΣADC

These should allow new (& more consistent?) kinematic cuts for net-p and net-q using kinematically-decoupled centralities allowing “the whole TPC” for the moments analyses.

To-do

Explore new net-p and net-q cuts sets that might result in a consistent story re: μ_B/T & T

Can I select the low-(Npos||Nneg) tails in net-q and trace the “sampling divergence”?

Also, Glauber for ZDC *vs.* BBC, 62 GeV & 39 GeV, mixed ratios, plus your suggestions...