Extracting $(\mu_B, T)$ from Cumulants of Multiplicity Distributions

M. Stephanov, Rice Workshop, May 23-25, 2012

In the NLSM, experimentally-measured moments products are proportional to powers of the correlation length (critical opalescence)

Divergent values may indicate the Critical Point

It has thus been popular to measure the shapes of multiplicity distributions, as quantified by the moments, $\mu, \sigma^2, S, K$, to search for the CP.

Decreased $1/VT^3$ dependence via $S\sigma = C_3/C_2$, $K\sigma^2 = C_4/C_2$

There is another analysis direction based on the multiplicity distribution shape information that can be pursued, and so far this direction is underexplored in STAR...

Use the cumulants to infer $(\mu_B, T)$...

Are our net-p and net-q results “consistent”?

Have we sculpted the net-p and net-q results via the different cuts sets that we use for each?
The “standard” approach to infer \((\mu_B, T)\) from a data sample involves statistical hadronization models, such as THERMUS.

S. Wheaton et al., Comp. Phys. Comm., 180, 84 (2009)

At one \(\sqrt{s_{\text{NN}}} \) & centrality:
Measure the ratios of efficiency-corrected average multiplicities \((C_1)\) of identified particles in a specific kinematic region \(|y|<0.1\) for light hadrons

Then assume a (grand, strangeness, micro) canonical ensemble.

That assumption then allows the fitting of the measured ratios to functions that have some free parameters:
\[
\mu_B, T, \gamma_S, \gamma_Q, R, \text{ etc} ...
\]

Applicability of the approach evaluated by \(\chi^2\), use of other ensembles, etc.
STAR SHM results

Centrality dependence in GCE vs. SCE vs. μCE?
Effects of wider rapidity gate?
We have very mature results on the net-p and net-q multiplicity distribution cumulants. Use these plus Lattice QCD to infer \((\mu_B, T)\).


Frithjof Karsch, University of Houston Colloquium, Sept. 24, 2013

**Determination of T and \(\mu_B\) from cumulant ratios**

- in thermal equilibrium any two ratios of cumulants should allow to fix temperature and baryon chemical potential

\[
R_{n,m}^X = \frac{\chi_{n,\mu}^X}{\chi_{m,\mu}^X}, \quad X = B, Q, S
\]

NLO Taylor expansion

- ratios with \(n+m\) even or odd show different sensitivity to \(T\) and \(\mu_B\)

\[
R_{12}^X \equiv \frac{M_X}{\sigma_X^2} = \frac{\mu_B}{T} \left( R_{12}^{X,1} + R_{12}^{X,3} \left( \frac{\mu_B}{T} \right)^2 + \mathcal{O}(\mu_B^4) \right),
\]

\[
R_{31}^X \equiv \frac{S_X \sigma_X^3}{M_X} = R_{31}^{X,0} + R_{31}^{X,2} \left( \frac{\mu_B}{T} \right)^2 + \mathcal{O}(\mu_B^4),
\]

\[
M_X \sim \chi_1^X : \quad \text{mean}
\]
\[
\sigma_X^2 \sim \chi_2^X : \quad \text{variance}
\]
\[
S_X \sim \chi_3^X / (\chi_2^X)^{3/2} : \quad \text{skewness}
\]

\(\blacklozenge\) if fluctuations are sensitive to equilibrium physics at a unique \((T, \mu_B)\) pair
(\(\mu_B, T\)) from Moments

Ratios of Multiplicity Distribution Cumulants: \(R_{xy} = C_x/C_y\)

\[ \frac{M_Q(\sqrt{s})}{\sigma_Q^2(\sqrt{s})} = \frac{\langle N_Q \rangle}{\langle (\delta N_Q)^2 \rangle} = \frac{\chi_1^Q(T, \mu_B)}{\chi_2^Q(T, \mu_B)} = R_{12}^{Q,1}(T)\mu_B + R_{12}^{Q,3}(T)\mu_B^3 + \ldots = \mathbb{R}_{12}^Q(T, \mu_B) \]

\[ \frac{S_Q(\sqrt{s})\sigma_Q^3(\sqrt{s})}{M_Q(\sqrt{s})} = \frac{\langle (\delta N_Q)^3 \rangle}{\langle N_Q \rangle} = \frac{\chi_3^Q(T, \mu_B)}{\chi_1^Q(T, \mu_B)} = R_{31}^{Q,0}(T) + R_{31}^{Q,2}(T)\mu_B^2 + \ldots = \mathbb{R}_{31}^Q(T, \mu_B) \]

\(\text{baryometer, fixes} \ \mu_B^f\)

\(\text{thermometer, fixes} \ \mu_T^f\)

LO linear in \(\mu_B\)

LO independent of \(\mu_B\)

**HIC**

- mean: \(M_Q\)
- variance: \(\sigma_Q^2\)
- skewness: \(S_Q\)
- \(\delta N_Q = N_Q - \langle N_Q \rangle\)

**LQCD**

- STAR, PHENIX

- STAR

- generalized charge susceptibilities:

\[ \chi_n^Q(T, \mu) = \frac{1}{VT^3} \frac{\partial^n \ln Z(T, \mu)}{\partial (\mu_Q/T)^n} \]
Basic approach: Measure $R_{12}$ and $R_{31}$, then pick off $\mu_B/T$ and $T$ from the Lattice results.

$$R^Q_{31} = S_Q \sigma_Q^3 / M_Q$$

$$R^Q_{12} = M_Q / \sigma_Q^2$$

<table>
<thead>
<tr>
<th>$S_Q \sigma_Q^3 / M_Q$</th>
<th>$T^f$ [MeV]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\geq 2$</td>
<td>$\leq 155$</td>
</tr>
<tr>
<td>$\sim 1.5$</td>
<td>$\sim 160$</td>
</tr>
<tr>
<td>$\leq 1$</td>
<td>$\geq 170$</td>
</tr>
</tbody>
</table>

BNL-BI: PRL 109, 192302 (2012)

<table>
<thead>
<tr>
<th>$M_Q / \sigma_Q^2$</th>
<th>$\mu_B^f / T^f$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01–0.02</td>
<td>0.1–0.2</td>
</tr>
<tr>
<td>0.03–0.04</td>
<td>0.3–0.4</td>
</tr>
<tr>
<td>0.05–0.08</td>
<td>0.5–0.7</td>
</tr>
</tbody>
</table>

for: $T^f \sim 160$ MeV
Results when using QM2012 net-q cumulants

Thermometer from $R_{13}^Q$

variation of $T^{f,ch}$ is < 5 MeV for $\sqrt{s_{NN}} > 19$ GeV as a first start, use the average value over $\sqrt{s_{NN}} = 19.6 - 200$ GeV

$\overline{R}_{31}^Q = 1.56(16)$

STAR preliminary Quak Matter 2012

$R_{31}^Q = S_Q \sigma_Q^3 / M_Q$

Baryometer from $R_{21}^Q$

$R_{12}^Q = M_Q / \sigma_Q^2$

S. Mukherjee, WWND 2013

Thermometer:
Reasonable agreement to lattice $T_c = 154(9)$ MeV

Baryometer:
Fair agreement of PHENIX data to HRG

$T^f = 158(7)$ MeV

$T_c(\mu_B=0) = 154(9)$ MeV
$T^{f,ch} = 160(5)$ MeV

$\sqrt{s}$ [GeV]: 62.4

$\mu_B^f / T^f$ agree reasonably with $\mu_B^{f,ch} / T^{f,ch}$
(\(\mu_B, T\)) from Moments

Thermodynamic consistency

The values of \((\mu_B, T)\) from \(R_{xy}^B\) should be consistent with those from \(R_{xy}^Q\)

S. Mukherjee, WWND 2013

![Graph showing the relationship between \(R_{12}^Q/R_{12}^B\) and \(\mu_B/T\) for different temperatures.]

If the fluctuations are described by equilibrium thermodynamics, \(R_{12}^Q\) and \(R_{12}^B\) must contain identical information regarding \(T\) and \(\mu_B\).

However ...

currently STAR preliminary @ \(\sqrt{s_{NN}} = 200\) GeV: \(R_{12}^Q/R_{12}^P \approx 0.06\)

a problem!!
give inconsistent values for $\mu_B^f$ a problem !!
The Wuppertal-Budapest LQCD group has also recently investigated this direction.

FIG. 4 (color online). $R_{12}^Q$ as a function of $\mu_B$: the different colors correspond to the continuum extrapolated lattice QCD results, calculated at different temperatures. The three points correspond to preliminary STAR data for $M_Q/\sigma_Q^2$ at different collision energies: $\sqrt{s} = 27, 39, 62.4$, from Ref. [6].


FIG. 5 (color online). $R_{31}^B$: the colored symbols correspond to lattice QCD simulations at finite $N_f$. The black points correspond to the continuum extrapolation.

TABLE I. Freeze-out baryon chemical potentials vs the corresponding collision energy of the three STAR measurements from Ref. [6]. The errors come from the uncertainty of the freeze-out temperature, the lattice statistics, and the experimental error.

<table>
<thead>
<tr>
<th>$\sqrt{s}$ [GeV]</th>
<th>$\mu_B^f$ [MeV]</th>
</tr>
</thead>
<tbody>
<tr>
<td>62.4</td>
<td>44(3)(1)(2)</td>
</tr>
<tr>
<td>39</td>
<td>75(5)(1)(2)</td>
</tr>
<tr>
<td>27</td>
<td>95(6)(1)(5)</td>
</tr>
</tbody>
</table>

$\mu_B^f = 0_{\text{lat}} 0_{\text{exp}}$
My goals

1. Use our latest efficiency-corrected results for $R_{xy}^{B,Q}$ to extract $(\mu_B, T)$
2. Produce new values of $R_{xy}^{B,Q}$ using different centrality definitions to allow more consistent kinematic acceptances for $R_{xy}^{B}$ and $R_{xy}^{Q}$

Now we have efficiency-corrected values! Effect of kinematic acceptance (different for net-p and net-q)?

F. Karsch, private communication

Do Not Circulate

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**net-p**
- refmult3corr ($\pi$&K, $|y|<1.0$)
- $|y|<0.5$, $0.4<P_T<0.8$
- $n_\sigma(p)<2$
- Nhitsfit>20, no Nhitsdedx cut
- DCAglobal<1
- $|ZvtxTPC-ZvtxVPD|<3$ ($\geq 39$ GeV)

**net-q**
- refmult2corr (chgd, $0.5<|\eta|<1$)
- $|\eta|<0.5$, $0.2<P_T<2.0$ -spallation p
- Nhitsfit>20, Nhitsdedx>10
- DCAglobal<1
- $|ZvtxTPC-ZvtxVPD|<4$ ($\geq 39$ GeV)
$R_{12} = \frac{C_1}{C_2}$, net-p, $|y|<0.5$, refmult3corr($|y|<1$)

Good sensitivity to $\mu_B/T$ for $\sqrt{s_{NN}} \geq 39$ GeV
$R_{31} = \frac{C_3}{C_1}$, net-$p$, $|y| < 0.5$, refmult3corr($|y| < 1$)

Very little $T$ dependence.
Large uncertainties when comparing to LQCD.
$R_{12} = C_1/C_2$, net-$q$, $|\eta| < 0.5$, refmult2corr(0.5$<|\eta|<1$)

Good sensitivity to $\mu_B/T$ for $\sqrt{s_{NN}} \geq 27$ GeV
(µ_B, T) from Moments

$R_{31} = \frac{C_3}{C_1}$, net-q, $|\eta| < 0.5$, refmult2corr(0.5 < $|\eta| < 1$)

Large uncertainties.
Data often outside LQCD-allowed area.
Cumulants+LQCD imply $\mu_B/T$ decreases as centrality decreases (similar to SHM w/ GCE)

$\mu_B/T$ from net-p and net-q diverge as $\sqrt{s_{NN}}$ decreases.

$\mu_B/T$ from net-p $> \mu_B/T$ from net-q

SHM results similar to the Cumulants+LQCD values (in between net-p & net-q)
Net-q values are all over the map w.r.t. LQCD allowed range...

Net-p values allow an extraction of T from R_{31}Q, but not with much sensitivity...
Summary so far:

Used latest efficiency-corrected net-p and net-q moments products to constrain $\mu_B/T$ & $T$ using LQCD predictions. This is an alternative to SHM approaches...

Reasonable sensitivity to $\mu_B/T$ from $R_{12}^{net-p}$ and $R_{12}^{net-q}$...

Not much sensitivity to $T$ from $R_{31}^{net-p}$...

Data for $R_{31}^{net-q}$ has large errors and are often outside the LQCD allowed range...

$\mu_B/T$ from $R_{12}^{net-p}$ & $R_{12}^{net-q}$ increases as the centrality increases...

Similar to the centrality dependence from the STAR SHM results with the GCE...

$\mu_B/T$ from $R_{12}^{net-p}$ & $R_{12}^{net-q}$ are inconsistent, and become more so as $\sqrt{s_{NN}}$ decreases...

Is this a result of the different kinematic cuts used in the net-p and net-q analyses?

<table>
<thead>
<tr>
<th>net-p</th>
<th>net-q</th>
</tr>
</thead>
<tbody>
<tr>
<td>refmult3corr ($\pi$&amp;K, $</td>
<td>y</td>
</tr>
<tr>
<td>$</td>
<td>y</td>
</tr>
<tr>
<td>$n_o(p)&lt;2$</td>
<td>Nhitsfit&gt;20, no Nhitsdedx cut</td>
</tr>
<tr>
<td>Nhitsfit&gt;20, no Nhitsdedx cut</td>
<td>DCAglobal&lt;1</td>
</tr>
<tr>
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</tr>
<tr>
<td>$</td>
<td>ZvtxTPC-ZvtxVPD</td>
</tr>
</tbody>
</table>

To explore this, I need centrality definitions that do not use the TPC...

...and/or should not strongly autocorrelate with the total multiplicity of TPC primaries...
With a centrality definition that does not use the TPC, I can measure the net-p and net-q cumulants using the same centrality definition (√), more similar kinematic cuts (√), and a less restrictive pseudorapidity range for net-q (√)...

Recall my earlier studies on alternative centrality definitions (2011):
http://wjllope.rice.edu/fluct/protected/cent_slides_20110817.pdf
http://wjllope.rice.edu/fluct/protected/cent_slides_20110914.pdf

At that time, I studied BEMC $\Sigma E$, and ZDC vs. BBC.

BEMC $\Sigma E$ showed some energy scale jumps in the low-$\sqrt{s_{NN}}$ BES data, but at the moment I am most interested in the 200 GeV data, where the peds and gains are in good shape.

Will explore:
- BBC $\Sigma ADC$
- BEMC $\Sigma E$
- BEMC $N_{\text{towers}}$ (ADC > pedavg+4*pedrms)
- ZDC $\Sigma ADC$ vs. BBC $\Sigma ADC$

Of course, this same code can also still use the “standard” refmulfXcorr and the same cuts used in the net-p and net-q papers to check the consistency, and I can also explore the sensitivity of the results to different cuts and centrality selections.
New !TPC centrality, 200 GeV

BEMC $\Sigma E$

BEMC $N_{\text{towers}}$

BBC $\Sigma ADC$

rejected w/ $\pm 5\sigma$ cut
Glauber calculation for BBC $\Sigma$ADC, 200 GeV

$\mu_B, T$ from Moments

Entries 4032810

$\Sigma$ADC, $m_{min}=100$, $n_{pp}=2.500$, $k=0.60$, $x=0.14$

consteff=0, eff=0.14 $\rightarrow$ Npt= 750, $\chi^2$/Npt= 9.60
Glauber calculations for BEMC variables, 200 GeV
{DS} unique identifier for year and $\sqrt{s_{NN}}$

- **Data**
- **Compiled C++ code**

**anpp:**
- select minbias trigger, apply $|Z_{vtx}|$ cut.
- calculate refmultX
- save event info and all primary tracks to TTrees

**fluct:**
- fill 4 “base” TH2Ds for specific track cut sets
  - (net,tot, pos, neg) vs. centrality variable

**mix:**
- read TH2Ds from net-p paper, net-q paper, or fluct
- calculate Cx, Rxy vs. centrality variable
- efficiency corrections
- CBW averaging
- bootstrap errors
- Sampled singles/IRV cumulant arithmetic

**qa:**
- bad runs: 30 variables, check 6, require $\geq 4$ vars fail
- bad events: 10 2D correlation plots, check 2, ±$N\sigma$ cuts

**fluctplot:**
- collect results from all sources and make final plots
- make connections to LQCD

BulkCorr PWG Meeting, 11/27/2013
(μₜ, T) from Moments

μₜ/T from fluct net-p and net-q

R_{12}^{net-p} is quite stable vs. centrality variable used, R_{12}^{net-q} is not...

fluct code reproduces net-p paper Cₓ and R_{12}, but not net-q paper Cₓ and R_{12}...
Used latest efficiency-corrected net-p and net-q moments products to constrain $\mu_B/T$ & $T$ using LQCD predictions. This is an alternative to SHM approaches...

- Reasonable sensitivity to $\mu_B/T$ from $R_{12}^{\text{net-p}}$ and $R_{12}^{\text{net-q}}$
- Not much sensitivity to $T$ from $R_{31}^{\text{net-p}}$
- Data for $R_{31}^{\text{net-q}}$ has large errors and are often outside the LQCD allowed range

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Similar to the centrality dependence from the STAR SHM results with the GCE

$\mu_B/T$ from $R_{12}^{\text{net-p}}$ & $R_{12}^{\text{net-q}}$ are inconsistent, and become more so as $\sqrt{s_{NN}}$ decreases...

There are two recent PRLs from two major LQCD collaborations, who will soon use the new efficiency-corrected net-p and net-q paper results to constrain $\mu_B/T$ & $T$

Aside from the CP search, do the two moments papers tell a consistent story at high $\sqrt{s_{NN}}$?

Four new centrality definitions based on:

- BBC $\Sigma_{\text{ADC}}$, BEMC $\Sigma_{\text{E}}$, BEMC $N_{\text{towers}}$, ZDC $\Sigma_{\text{ADC}}$ vs. BBC $\Sigma_{\text{ADC}}$

These should allow new (& more consistent?) kinematic cuts for net-p and net-q using kinematically-decoupled centralities allowing “the whole TPC” for the moments analyses.

To-do

- Explore new net-p and net-q cuts sets that might result in a consistent story re: $\mu_B/T$ & $T$
- Can I select the low-$(N_{\text{pos}}||N_{\text{neg}})$ tails in net-q and trace the “sampling divergence”?
- Also, Glauber for ZDC vs. BBC, 62 GeV & 39 GeV, mixed ratios, plus your suggestions...