

## Impact parameter dependence of the disappearance of transverse flow in nuclear collisions

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(Received 5 June 1995)

The impact parameter dependence of the disappearance of directed transverse flow has been investigated for  $^{40}\text{Ar}+^{45}\text{Sc}$  reactions using the Michigan State University  $4\pi$  Array upgraded with the High Rate Array (HRA). The energy at which collective transverse flow in the reaction plane disappears, the balance energy ( $E_{\text{bal}}$ ), is found to increase approximately linearly as a function of impact parameter. Comparison of our measured values of  $E_{\text{bal}}(b)$  shows agreement with predictions of Quantum Molecular Dynamics (QMD) model calculations.

PACS number(s): 25.70.Pq

The study of collective flow in nucleus-nucleus collisions can provide information about the nuclear equation of state (EOS) [1–4]. Collective transverse flow in the reaction plane disappears at an incident energy, termed the balance energy ( $E_{\text{bal}}$ ) [5], where the attractive scattering dominant at energies around 10 MeV/nucleon balances the repulsive interactions dominant at energies around 400 MeV/nucleon [6–15]. We have recently completed a systematic study of the disappearance of flow for central collisions in symmetric entrance channels, which showed that  $E_{\text{bal}}$  scales as  $A^{-1/3}$  where  $A$  is the mass of the combined projectile-target system [16]. The general trend of this result, which was reproduced by the Boltzmann-Uehling-Uhlenbeck model [16] and the Landau-Vlasov model [17] calculations at a fixed impact parameter, demonstrated that  $E_{\text{bal}}$  is insensitive to the compressibility of the EOS but sensitive to the in-medium nucleon-nucleon cross section.

The importance of the role of the impact parameter in the determination of the disappearance of flow has been recognized [7,10,18]. As two nuclei collide, the pressure and density increase in the interaction region. At nonzero impact parameters there is anisotropy in the pressure, resulting in a transverse flow of nuclear matter in the directions of lowest pressure. In symmetric collisions the compressed midrapidity participant volume is expected to decrease in size with increasing impact parameter, so that a larger incident energy is required to compensate for the effects of the mean field in more peripheral collisions [18]. Using a transverse momentum analysis method [19], we show that flow can be determined from midrapidity participant fragments for relatively peripheral collisions. The impact parameter dependence of the balance energies extracted from the measured flow values agrees with predictions from Quantum Molecular Dynamics (QMD) model calculations.

The present measurements were carried out with the Michigan State University  $4\pi$  Array [20] at the National Superconducting Cyclotron Laboratory (NSCL) using beams from the K1200 cyclotron. A target of 1.0 mg/cm<sup>2</sup> Sc was

bombarded with  $^{40}\text{Ar}$  projectiles ranging in energy between 35 and 115 MeV/nucleon in 10 MeV/nucleon steps. Beam intensities were approximately 100 electrical pA. Prior to this experiment, the MSU  $4\pi$  Array was upgraded with the High Rate Array (HRA). The HRA is a close-packed pentagonal configuration of 45 phoswich detectors spanning laboratory polar angles  $3^\circ \leq \theta \leq 18^\circ$ . With the HRA we obtained  $Z$  resolution up to the charge of the  $^{40}\text{Ar}$  projectile, and mass resolution for the hydrogen isotopes. The main ball of the MSU  $4\pi$  Array consists of 55 Bragg curve counters followed by 170 phoswich detectors covering the angles  $18^\circ \leq \theta \leq 162^\circ$ . Data were taken with a minimum bias trigger that required at least one hit in the HRA (HRA-1 data), and a more central trigger where at least two hits in the main ball (Ball-2 data) were required. The flow analysis described below was performed with the Ball-2 data as done in Ref. [16].

We use a transverse momentum analysis method [19] in which the impact parameter and the orientation of the reaction plane must be determined. The impact parameter  $b$  of each event is assigned through cuts on centrality variables [21] measured with the improved acceptance of the upgraded MSU  $4\pi$  Array. The centrality variable chosen here was the reduced transverse kinetic energy of each event  $\hat{E}_t$ , as defined in Ref. [22]. Using methods similar to those detailed in Ref. [23],  $\hat{E}_t$  is found to be an appropriate variable to use as a centrality filter for this system over the range of beam energies studied, and does not autocorrelate with the flow observables. The reaction plane is calculated using the method of azimuthal correlations [24], which is a good method to determine the reaction plane in cases where transverse collective motion can become weak (e.g., beam energies near the balance energy).

As an example of the method used for impact parameter selection, events with  $\hat{E}_t$  in the top 10% of the inclusive  $\hat{E}_t$  spectrum for the Ball-2 data were assigned to the most central bin. This corresponds to a reduced impact parameter of  $\hat{b} = (b/b_{\text{max}}) \leq 0.32$  as calculated through a simple geomet-

TABLE I. Reduced impact parameter bins. The values of  $\hat{b}$  correspond to the upper limit of each bin.

Bin No.	Cut on $\hat{E}_t$	Geometric $\hat{b}$	Corrected $\hat{b}$
BIN1	Top 10%	0.32	0.28
BIN2	10–20 %	0.45	0.39
BIN3	20–30 %	0.55	0.48
BIN4	30–40 %	0.63	0.56
BIN5	40–50 %	0.71	0.62
BIN6	50–75 %	0.87	0.76
BIN7	Bottom 25%	1.00	0.88

ric prescription [21], where  $b_{\max}$  represents the largest impact parameter leading to a triggered event. If the measured cross section was equivalent to the geometric (hard sphere) cross section, then  $b_{\max}$  would be the sum of the projectile and target radii  $R_{\text{proj}} + R_{\text{targ}}$ . However, the actual maximum impact parameter to trigger an event is less than  $R_{\text{proj}} + R_{\text{targ}}$ , due to hardware trigger bias and detector acceptance. In order to estimate  $b_{\max}$  for the Ball-2 data, we adjusted the overall normalization of the inclusive  $\hat{E}_t$  spectrum to fit the same distribution for data taken with the less selective HRA-1 trigger. From the ratio of the cross sections represented by the two distributions we extracted for the Ball-2 events a value of  $b_{\max} = 0.88 \pm 0.04 (R_{\text{proj}} + R_{\text{targ}})$ , under the assumption that  $R_{\text{proj}} + R_{\text{targ}}$  is the largest impact parameter to trigger an HRA-1 event. This results in a corrected  $\hat{b} \leq 0.28$  for the top 10% most central Ball-2 events. The correction factor did not vary significantly over the range of beam energies we measured. The remaining impact parameter bins and the corresponding reduced impact parameters in the simple geometric picture are summarized in Table I. Also listed in this table are the effective values of the reduced impact parameter corrected for bias due to the hardware trigger condition.

After the impact parameter of the event has been assigned, the reaction plane is calculated using the method of azimuthal correlations [24]. First a particle of interest (POI) is chosen from the event. Autocorrelation is avoided by omitting this POI in the calculation of the reaction plane [19]. The momenta of the remaining particles are projected into a plane perpendicular to the beam axis (taken as the origin in this plane). A line passing through the origin is then simultaneously fit to the transverse momentum coordinates of these fragments. The azimuthal angle of this line becomes the azimuthal angle of the reaction plane. The positive half of the reaction plane is associated with the side on which the total transverse momentum in the reaction plane is greatest. Finally, the POI's transverse momentum in the reaction plane  $p_x$  is evaluated by projecting it into this calculated reaction plane. This procedure is repeated for each particle in the event for all events with at least four identified particles.

In Fig. 1 we show the mean transverse momentum in the reaction plane  $\langle p_x \rangle$  plotted versus the reduced center-of-mass (c.m.) rapidity  $(y/y_{\text{proj}})_{\text{c.m.}}$ . The data are for He fragments from 55 MeV/nucleon  $^{40}\text{Ar} + ^{45}\text{Sc}$  reactions at four different reduced impact parameter bins (as listed in Table I). The errors shown in each panel are statistical. The data are fit with a straight line over the midrapidity region

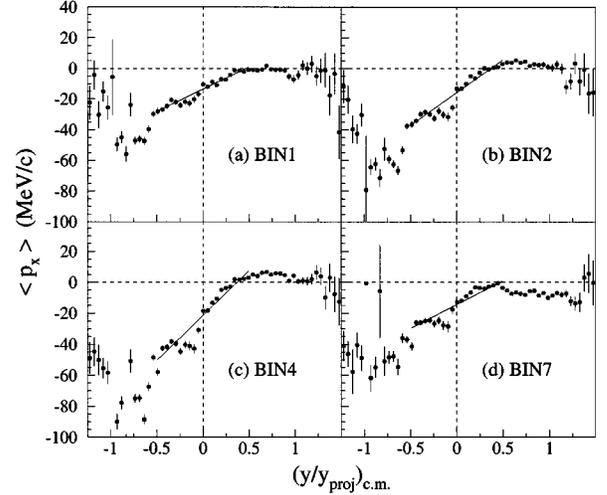


FIG. 1. Mean transverse momentum in the reaction plane versus the reduced rapidity in the center-of-mass frame for  $Z = 2$  fragments in 55 MeV/nucleon  $^{40}\text{Ar} + ^{45}\text{Sc}$  reactions. The reduced impact parameter bins, as indicated in each panel, are listed in Table I. The straight lines are fit in the region  $-0.5 \le (y/y_{\text{proj}})_{\text{c.m.}} \le 0.5$ .

$-0.5 \le (y/y_{\text{proj}})_{\text{c.m.}} \le 0.5$  and the slope of this line is defined as the directed transverse flow. The data exhibit the characteristic “S-shape” associated with collective transverse flow in the reaction plane. The offsets from the origin occur because no recoil correction was applied in the reaction plane calculation for this analysis. We found that a constant fraction of the system mass could not be used in the recoil correction, as defined in Ref. [9], to make the offsets vanish for all impact parameters at a given beam energy. As the impact parameter increases in Fig. 1, the transverse flow increases, passes through a maximum, and diminishes for the most peripheral impact parameter bin shown. This behavior is in qualitative agreement with previous results that range in beam energy from 55 MeV/nucleon [25] to 400 MeV/nucleon [2]. That collective transverse flow is maximal at some intermediate impact parameter is reasonable because it must vanish at the extrema, i.e., for grazing and perfectly central collisions.

The extracted values of the transverse flow plotted versus the beam energy are shown in Fig. 2 for the four most central reduced impact parameter bins (as listed in Table I). The errors shown are the statistical errors on the slopes of the linear fits (the systematic error associated with the range of the fitting region is  $+3$  MeV/c and  $-1$  MeV/c). The data points for each  $\hat{b}$  bin are fit with a second-order polynomial for the purpose of finding the balance energy  $E_{\text{bal}}$ . As in Ref. [5] we found that the analytic form of the fitting function does not significantly affect the value of the extracted balance energy. We assume collective transverse flow to be symmetric in the vicinity of the balance energy, and our measurements are unable to distinguish the sign (+ or -) of the flow, so that a parabolic function is the lowest order symmetric fit we can use without *a priori* knowledge of  $E_{\text{bal}}$ . In addition this local parabolic fit, also investigated in Ref. [5], facilitates extraction of the balance energy for the larger impact parameters where the flow does not strongly reappear.

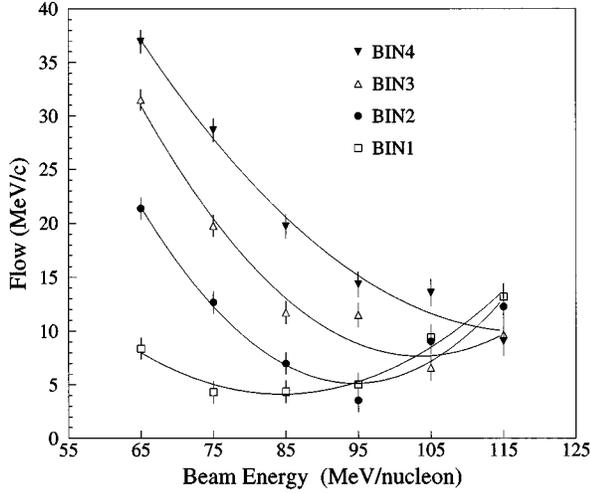


FIG. 2. Excitation functions of the measured transverse flow in the reaction plane for  $Z = 2$  fragments at four reduced impact parameter bins for  $^{40}\text{Ar} + ^{45}\text{Sc}$  reactions. The corresponding values of  $\hat{b}$  are given in Table I. The solid curves are parabolic fits as described in the text.

The curves shown in Fig. 2 pass through minima for which the value of the abscissa corresponds to the balance energy at each reduced impact parameter  $E_{\text{bal}}(b)$ . The curves do not pass through zero at  $E_{\text{bal}}(b)$  because no recoil correction was used in the reaction plane determination. Although the extracted values of  $E_{\text{bal}}(b)$  remain unaffected within error, the recoil correction was found to shift the locus of the data for a given  $\hat{b}$  bin vertically downward, and even cause negative flow values, which is inconsistent with the basic premises of the transverse momentum analysis [19]. For the

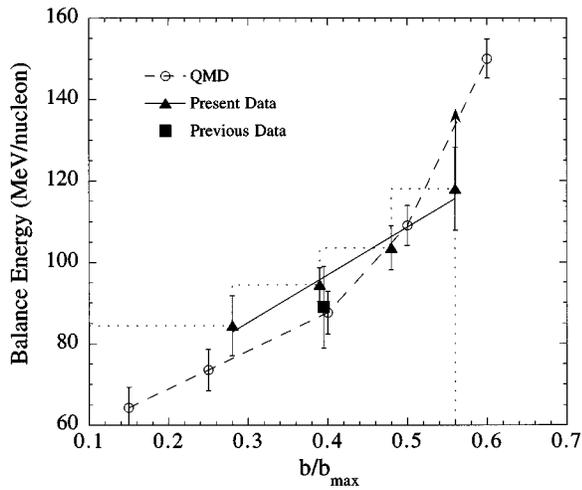


FIG. 3. Measured balance energies for  $^{40}\text{Ar} + ^{45}\text{Sc}$  reactions at the four most central reduced impact parameter bins compared with the predictions of the QMD model for  $^{40}\text{Ca} + ^{40}\text{Ca}$  reactions [26]. The experimental values of  $E_{\text{bal}}(b)$  are plotted at the upper limit of each  $\hat{b}$  bin represented by the dotted histogram. The curves are included only to guide the eye. The value of previous data is from Ref. [16].

TABLE II. Measured values of the balance energies for  $^{40}\text{Ar} + ^{45}\text{Sc}$  reactions extracted for the four most central reduced impact parameter bins. The errors listed are statistical.

Corrected $\hat{b}$	$E_{\text{bal}}$ (MeV/nucleon)
0.28	$84 \pm 7$
0.39	$95 \pm 4$
0.48	$104 \pm 5$
0.56	$119 \pm 10$

largest  $\hat{b}$  bin displayed only a lower limit on the value of  $E_{\text{bal}}(b)$  could be determined from these data, because the higher beam energies necessary to extract  $E_{\text{bal}}(b)$  for more peripheral collisions were not available from the K1200 cyclotron. The horizontal shift in the minima of the curves in Fig. 2 clearly indicates that  $E_{\text{bal}}(b)$  increases as the impact parameter increases. This result is in qualitative agreement with Refs. [10,15], but here we are able to more definitively extract  $E_{\text{bal}}(b)$  for larger impact parameters because our measurements include more data points above the balance energy. In Ref. [15] this result was found through an entirely different analysis using correlation functions, which does not require reaction plane determination or recoil correction.

Transport model calculations can incorporate soft and stiff descriptions of the nuclear EOS as well as momentum dependence in the mean field. The predictions of the Quantum Molecular Dynamics (QMD) model [26] calculations are displayed in Fig. 3 for a stiff equation of state without momentum dependence for  $^{40}\text{Ca} + ^{40}\text{Ca}$  reactions (open circles). These points are calculated for a fixed impact parameter and are not corrected for the acceptance effects of our detector array. Also shown in this figure are the measured values of the balance energies for  $^{40}\text{Ar} + ^{45}\text{Sc}$  reactions extracted for the four most central reduced impact parameter bins (solid triangles). These experimental values of  $E_{\text{bal}}(b)$  are plotted at the upper limit of each  $\hat{b}$  bin represented by the dotted histogram. The errors shown on the measured values of the balance energies are statistical (the systematic error is estimated to be +5% and -0%). We find that  $E_{\text{bal}}(b)$  increases approximately linearly as a function of the impact parameter in good agreement with Ref. [26]. This agreement demonstrates that the impact parameter dependence of the disappearance of transverse flow may potentially provide a powerful probe of the nuclear EOS. The result shown for BIN2 ( $\hat{b} = 0.39$ ) is comparable with our previous measurement of  $E_{\text{bal}}$  for  $^{40}\text{Ar} + ^{45}\text{Sc}$  of  $87 \pm 12$  MeV/nucleon (solid square) at  $\hat{b} = 0.40$  assigned through a cut on the total transverse momentum [16]. The value of  $\hat{b}$  for this point has not been corrected as in the present analysis. The measured values of the balance energies for  $^{40}\text{Ar} + ^{45}\text{Sc}$  reactions extracted for the four most central reduced impact parameter bins are listed in Table II.

In summary, we have investigated the impact parameter dependence of the disappearance of directed transverse flow for  $^{40}\text{Ar} + ^{45}\text{Sc}$  reactions using the MSU  $4\pi$  Array upgraded with the HRA. Our results indicate that the balance energy increases approximately linearly as a function of impact parameter. Physically this dependence results from a smaller participant zone in more peripheral collisions, so that a larger

incident energy is required to overcome effects of the mean field. Comparison of the trends in our measured values of  $E_{\text{bal}}(b)$  is consistent with the predictions of QMD model calculations. We agree with the point of view expressed in Ref. [26] that the balance energy is indeed dependent upon impact parameter.

We gratefully acknowledge the technical assistance of D. Swan and J. Wagner during the design and assembly stages of the HRA. Comments and suggestions from S. Soff and H. Stöcker were most appreciated. This work was supported by the National Science Foundation under Grant Nos. PHY-92-14992 (NSCL/MSU) and PHY-92-11611 (SUNY).

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